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Coupled transport of heat and mass in laminar tube flow

Lung-mau Huang

COUPLED TRANSPORT OF HEAT AND MASS

IN LAMINAR TUBE FLOW

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Tu-Shan Chen (Adviser) Lyle G. Rhea

M. E. Findlay

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ABSTRACT

The object of this investigation is to study the mutual dependence of heat and mass transport in laminar and hydrodynamically developed tube flow. Consideration is given to gas flow through a tube whose wall is coated with a sublimable material so that heat and mass transfers take place between the wall and the flowing gas stream. The tube wall is thermally insulated from the external environment, with the result that the latent heat required by the sublimation process at the wall is supplied solely by convective transfer from the gas.

The analysis of the problem requires simultaneous solution of the energy and diffusion equations with couplings provided by the boundary conditions. An analytical solution is facilitated by employing a linear saturation state relationship for the vapor of the subliming solid. The resulting eigenvalue problem does not belong to the conventional Sturm-Liouville system. A special integral relation is derived which serves as an orthogonality condition.

Mathematical expressions are derived and numerical results are presented for the streamwise variations of the bulk temperature and bulk mass fraction, of the wall temperature and mass fraction, of the wall heat and mass transfer rates, and of the local Nusselt number for nine different cases. Representative temperature and mass fraction profiles are also presented. Entrance lengths which characterize the near approach to fully developed conditions are tabulated. Comparisons of typical results are made with the case of coupled transport in a parallel-plate channel.

CHAPTER 1

INTRODUCTION

Heat transfer processes in duct and tube flows have been studied analytically for many years. A detailed account of this subject matter can be found, for example, in the book by Kays (1). In recent years, considerable attention has also been given to the analysis of mass transfer processes in internal flows (see, for example, references 2 and 3). However, a survey of the literature reveals that relatively little work has been done in connection with internal flow problems in which heat and mass transfer processes act simultaneously so that the heat transfer rate and the mass transfer rate are mutually dependent. References 4 to 6 treat problems of combined heat and mass transfer processes in laminar duct flows. The present research is also concerned with such a coupled transport process.

Specific consideration is given to flow of a gas in a circular tube whose wall is coated with a sublimable material. The tube wall is thermally isolated from the external environment. When the gas stream entering the tube is not saturated with the vapor of the sublimable material, sublimation will occur at the tube wall, the latent heat of sublimation being supplied by means of convective transport from the flowing gas to the wall. It is thus apparent that the rates of heat and mass transfer are mutually dependent. The mass fraction of the sublimed vapor will increase in the streamwise direction, while the gas temperature will decrease. At sufficiently large downstream distance, the mass fraction and the temperature approach cross-sectionally uniform, fully developed values.

The flow is assumed to be laminar and hydrodynamically developed, whereas the temperature and mass fraction are uniform across the inlet cross section. For the purpose of computational convenience, it will be assumed that a slug flow condition prevails inside the tube.

The formulation of the problem requires simultaneous consideration of the energy and diffusion equations, with coupling provided by the boundary conditions. At the tube wall, in addition to the energy balance between the rate of heat transfer and the latent heat of sublimation, the vapor-solid saturation condition is applied. Moreover, in order to facilitate an analytical solution of the problem, a linearized saturation state relationship is employed. This linear model has been found to be satisfactory (4).

The governing equations and the boundary conditions give rise to an eigenvalue problem wherein the energy and diffusion equations are found to share a common set of eigenvalues, but have different eigenfunctions. The mathematical system which stems from the present problem does not belong to the conventional Sturm-Liouville family. It is, therefore, necessary to derive a special integral condition to serve as an orthogonality relation.

Mathematical expressions are derived for various quantities of engineering interest. Numberical results are obtained and presented for the axial distributions of the bulk temperature and mass fraction, of the wall temperature and mass fraction, of the wall heat and mass fluxes, and of the local heat transfer coefficient for nine different cases. Representative temperature and mass fraction profiles are also presented. The length of the development region is deduced from the bulk temperature and mass fraction results characterizing the near approach to the fully developed conditions. Finally, comparisons are made with representative results from the slug flow solution in a parallel-plate channel (4).

CHAPTER 2

ANALYSIS AND SOLUTION

2-1. Formulation of the Problem

Consider a circular tube whose inner wall is coated with a sublimable material, while its outer wall is insulated so that no heat exchange takes place with the external environment. A hydrodynamically developed flow of gas enters the tube with a uniform temperature T_o. The entering gas may contain vapor of the sublimable material in the amount C_o which is less than the saturation value corresponding to the temperature T_o and the total pressure.

The starting point of the analysis is the conservation equations for energy and diffusion. These equations may be simplified by making the following assumptions:

(1) The radial velocity component v is negligibly small compared to the axial velocity component u. In fact, it has been estimated (4) that the radial velocity at the tube wall is about 0.01 percent of the average velocity in many sublimation problems (such as air-ice, air-naphthalene systems).

(2) The change of the rate of mass flow between the inlet section and the fully developed region is negligible because of the small sublimation rate. This rate of change of mass flow is on the order of a few percent (5).

(3) The fluid properties are constant. This assumption is based on the fact that the mass fraction of the sublimed vapor is very small.

(4) The axial heat conduction and axial mass fraction gradients are negligible; that is, the Peclet numbers for heat transfer Ur_o/α and diffusion Ur_o/D are very large.

In light of assumptions (1) to (4), the pertinent conservation equations for energy and diffusion can be written as

$$u\frac{\partial T}{\partial x} = \alpha\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \qquad u\frac{\partial C}{\partial x} = D\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right). \qquad (1)$$

In the above equations, T is the fluid temperature and C is the mass fraction of the vapor. The coordinate system is taken such that x measures the axial distance from the tube inlet and r measures the radial distance from the centerline of the tube. The radius of the tube is r_o.

Equations (1) can be recast into dimensionless form by employing the definitions

$$\eta = \frac{r}{r_o}, \qquad \xi = \frac{x/r_o}{Ur_o/\alpha}, \qquad \theta = \frac{T - T_f}{T_o - T_f}, \qquad \phi = \frac{C - C_f}{C_o - C_f} \qquad (2)$$

where T_f and C_f represent, respectively, the fluid temperature and the vapor mass fraction in the fully developed region. The denominator of the ξ variable is the Peclet number, Ur_o/α ; U is the average flow velocity and α is the thermal diffusivity.

With the foregoing dimensionless variables, equations (1) become

$$\frac{u}{U}\frac{\partial\theta}{\partial\xi} = \frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\theta}{\partial\eta}\right), \qquad \frac{u}{U}\frac{\partial\phi}{\partial\xi} = \frac{1}{L}\frac{1}{\eta}\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\phi}{\partial\eta}\right) \qquad (3)$$

where $L = \alpha/D$ denotes the Lewis number and D is the binary diffusion coefficient. The slug flow profile, i.e., $u/U = 1$, will be employed later.

The boundary conditions will now be examined. Since the tube wall is thermally isolated from the external environment, the latent heat for the sublimation process must be supplied solely by heat transfer from the fluid to the wall. The energy balance at the wall gives $q_w = \lambda_s \dot{m}$ (λ_s is the latent heat of sublimation and $\dot{m}$ is the local sublimation rate per unit area). In general, the mass flux $\dot{m}$ at the wall includes both convective and diffusive components (7). However, because of the small sublimation rates being considered, the former is very much smaller than the latter, and the convection contribution can be neglected. Then, upon noting that $q_w = -k(\frac{\partial T}{\partial r})_{r=r_o}$ from Fourier's law and that $\dot{m} = \rho D(\partial C/\partial r)_{r=r_o}$ from Fick's law, the energy balance at the wall $r = r_o$ (i.e., $\eta = 1$) yields

$$\frac{\partial C}{\partial r} = -\frac{k}{\rho D \lambda_s} \frac{\partial T}{\partial r} \text{ or } \frac{\partial \phi}{\partial \eta} = L[\frac{c_p}{\lambda_s}(\frac{T_o - T_f}{C_f - C_o})] \frac{\partial \theta}{\partial \eta} \text{ at } \eta = 1 \cdot \quad (4)$$

It can be shown that the quantity in braces is equal to unity, within the framework of the analysis. For this purpose, one takes a control volume which spans the tube cross section and extends from the entrance $x = 0$ into a location $x = x^*$ in the fully developed region. The temperature and mass fraction are uniform at both $x = 0$ and $x = x^*$. For small sublimation rates, the changes in the rate of mass flow can be neglected. Upon integrating each of equations (1) over the tube cross section and along the tube length from $x = 0$ to $x = x^*$, there results

$$\dot{M}c_p(T_f - T_o) = 2\pi r_o \int_0^{x^*} k\left(\frac{\partial T}{\partial r}\right)_{r_o} dx,$$

$$\dot{M}(C_f - C_o) = 2\pi r_o \int_0^{x^*} \rho D\left(\frac{\partial C}{\partial r}\right)_{r_o} dx \tag{5}$$

where $\dot{M} = \rho\pi r_o^2 U$ is the mass rate of flow. From equations (5) and the first of equations (4), it follows that

$$T_o - T_f = \frac{\lambda_s}{c_p}(C_f - C_o). \tag{6}$$

With the aid of equation (6), the boundary condition (4) becomes

$$\frac{\partial\phi}{\partial\eta} = L\frac{\partial\theta}{\partial\eta} \quad \text{at } \eta = 1. \tag{7}$$

Another boundary condition is that at every location x on the wall a saturation state for the solid-vapor system prevails. This gives rise to a unique relationship between the local vapor pressure and temperature at the wall; that is, $C_w = f(T_w)$, where $f(T_w)$ is a known function. In order to achieve an analytical solution, it is necessary to linearize the problem. For this purpose, the following linearized saturation-state relation is introduced:

$$C = aT + b. \tag{8}$$

A procedure for evaluating the constants a and b from given thermodynamic data and the numerical tests of the utility of equation (8) are detailed in reference 4. On the basis of the findings in reference 4, it can be concluded that equation (8) is a satisfactory representation for the saturation state relation. When equation (8) is applied at the wall and at the fully developed region, one has

$$C_w = aT_w + b, \qquad C_f = aT_f + b.$$

Thus,

$$C_w - C_f = a(T_w - T_f). \tag{9}$$

Next, if use is made of equations (2) and (6), there follows

$$\phi_w = -\left(\frac{a\lambda_s}{c_p}\right)\theta_w \qquad \text{at } \eta = 1. \tag{10}$$

The other boundary conditions specify the symmetry conditions at the centerline of the tube and the entrance conditions. These are

$$\frac{\partial\theta}{\partial\eta} = \frac{\partial\phi}{\partial\eta} = 0 \qquad \text{at } \eta = 0 \tag{11}$$

and

$$\theta = \phi = 1 \qquad \text{at } \xi = 0. \tag{12}$$

The mathematical system for the problem to be solved now consists of the conservation equations (3) subject to the boundary conditions (7), (10), (11), and (12).

2-2. Solution

The slug flow profile $u/U = 1$ will now be assumed in carrying out the solution. The elemental solutions of equations (3) can be obtained by the separation of variables method. They are

$$\theta = C_1 e^{-\beta^2 \xi} \Theta(\eta), \qquad \phi = C_2 e^{-\gamma^2 \xi} \Phi(\eta) \tag{13}$$

where β and γ are eigenvalues and Θ and Φ obey .

$$\frac{d}{d\eta}(\eta \frac{d\Theta}{d\eta}) + \eta\beta^2\Theta = 0, \qquad \frac{d}{d\eta}(\eta \frac{d\Phi}{d\eta}) + \eta L\gamma^2\Phi = 0. \tag{14}$$

The solutions of equations (14) can easily be found to be

$$\Theta = C_3 J_o(\beta\eta) + C_4 Y_o(\beta\eta),$$

$$\Phi = C_5 J_o(\gamma L^{\frac{1}{2}}\eta) + C_6 Y_o(\gamma L^{\frac{1}{2}}\eta) \tag{15}$$

where J_o and Y_o are, respectively, Bessel functions of the first and second kinds of zeroth order and the C's are constants. Applying the boundary conditions (11), one finds that $C_4 = C_6 = 0$. Thus, equations (13) reduce to

$$\theta = Ae^{-\beta^2 \xi}\Theta(\eta), \qquad \phi = Be^{-\gamma^2 \xi}\Phi(\eta) \tag{16}$$

$$\Theta = J_o(\beta\eta), \qquad \Phi = J_o(\gamma L^{\frac{1}{2}}\eta) . \tag{17}$$

Next, by imposing the boundary conditions (7) and (10), it can be readily demonstrated that

$$\beta = \gamma . \tag{18}$$

Thus, the energy and diffusion equations have the same eigenvalues, but have different eigenfunctions Θ and Φ.

A further application of the boundary conditions (7) and (10) yields

$$\frac{B}{A} = -(\frac{a\lambda_s}{c_p}) \frac{J_o(\beta)}{J_o(\beta L^{\frac{1}{2}})}, \qquad \frac{B}{A} = L^{\frac{1}{2}} \frac{J_1(\beta)}{J_1(\beta L^{\frac{1}{2}})} . \tag{19}$$

Elimination of the constants A and B results in

$$-\left(\frac{a\lambda_s}{c_p}\right) \frac{J_o(\beta)}{J_o(\beta L^{\frac{1}{2}})} = L^{\frac{1}{2}} \frac{J_1(\beta)}{J_1(\beta L^{\frac{1}{2}})} . \tag{20}$$

Where J_1 is the Bessel function of the first kind of first order. The eigenvalues β may be determined from equation (20). Inspection of the equation reveals that β depends on two parameters, the property group $a\lambda_s/c_p$ and the Lewis number L. For fixed values of L and $a\lambda_s/c_p$, a succession of roots of equation (20) may be found. In the present investigation, the eigenvalues were determined numerically by a root finder.

Owing to the linearity of the problem, a solution for θ and ϕ corresponding to fixed values of the parameters may be obtained by summing over all successive eigenvalues; that is,

$$\theta = \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 \xi} \Theta_n(\eta), \qquad \phi = \sum_{n=1}^{\infty} B_n e^{-\beta_n^2 \xi} \Phi_n(\eta). \tag{21}$$

The series coefficients A_n and B_n still remain to be determined. For this purpose, the conditions (12) at the tube inlet are employed. Thus,

$$1 = \sum_{n=1}^{\infty} A_n \Theta_n(\eta), \qquad 1 = \sum_{n=1}^{\infty} B_n \Phi_n(\eta). \tag{22}$$

To find the series coefficients A_n and B_n, one needs an orthogonality relationship involving the eigenfunctions Θ_n and Φ_n. For a conventional entrance region problem, the standard Sturm-Liouville orthogonality relation can be used. However, owing to the coupling between eigenfunctions, it is necessary to derive an integral condition which serves as an orthogonality relation.

One begins by applying the first of the equations (14) (i.e., the θ equation) for two distinct eigenvalues β_i and β_j. By conventional manipulations one derives

$$(\beta_i^2 - \beta_j^2) \int_0^1 \eta \theta_i \theta_j d\eta = \theta_i(1)\, \theta_j'(1) - \theta_j(1)\, \theta_i'(1). \quad (23)$$

Similarly, from the second of equations (14) applied for β_i and β_j,

$$L(\beta_i^2 - \beta_j^2) \int_0^1 \eta \Phi_i \Phi_j d\eta = \Phi_i(1) \Phi_j'(1) - \Phi_j(1)\, \Phi_i'(1). \quad (24)$$

Next, by introducing the boundary conditions (19), equation (24) can be rephrased as

$$-\frac{B_i B_j}{A_i A_j}\left(\frac{c_p}{a\lambda_s}\right) (\beta_i^2 - \beta_j^2) \int_0^1 \eta \Phi_i \Phi_j d\eta = \theta_j'(1)\theta_i(1) - \theta_i'(1)\theta_j(1) . \quad (25)$$

It may be observed that the right-hand sides of the equations (25) and (23) are identical. Upon eliminating this identical group, there follows

$$(\beta_i^2 - \beta_j^2)\left[\int_0^1 (A_i A_j \eta \theta_i \theta_j + \frac{c_p}{a\lambda_s} \eta B_i B_j \Phi_i \Phi_j) d\eta\right] = 0 . \quad (26)$$

When $\beta_i^2 \neq \beta_j^2$, the integral must be zero, and this leads to the orthogonality condition

$$\int_0^1 (A_i A_j \theta_i \theta_j + \frac{c_p}{a\lambda_s} B_i B_j \Phi_i \Phi_j) \eta d\eta = 0 . \quad (27)$$

Having determined the orthogonality condition, one can now proceed to find the coefficients A_n and B_n. This is done in the following manner. One multiplies the first of equations (22) by $\eta A_m \Theta_m$ and the second of equations (22) by $(\frac{c_p}{a\lambda_s})\eta B_m \Phi_m$, adds the thus-multiplied equations, and integrates over the range from $\eta = 0$ to $\eta = 1$. This procedure gives rise to the relation

$$\sum_{n=1}^{\infty} \int_0^1 (A_m A_n \eta \Theta_m \Theta_n + \frac{c_p}{a\lambda_s} B_m B_n \eta \Phi_m \Phi_n) d\eta$$

$$= \int_0^1 \eta A_m \Theta_m d\eta + \frac{c_p}{a\lambda_s} \int_0^1 \eta B_m \Phi_m d\eta . \tag{28}$$

After making use of the orthogonality condition, equation (27), there results

$$A_n^2 \int_0^1 \eta \Theta_n^2 d\eta + B_n^2 (\frac{c_p}{a\lambda_s}) \int_0^1 \eta \Phi_n^2 d\eta = A_n \int_0^1 \eta \Theta_n d\eta$$

$$+ B_n (\frac{c_p}{a\lambda_s}) \int_0^1 \eta \Phi_n d\eta . \tag{29}$$

Next, B_n may be eliminated with the aid of the first of equations (19). With this substitution, the expression for A_n reduces to

$$A_n = \frac{\int_0^1 \eta \Theta_n d\eta - \frac{J_o(\beta_n)}{J_o(\beta_n L^{\frac{1}{2}})} \int_0^1 \eta \Phi_n d\eta}{\int_0^1 \eta \Theta_n^2 d\eta + \frac{a\lambda_s}{c_p} [\frac{J_o(\beta_n)}{J_o(\beta_n L^{\frac{1}{2}})}]^2 \int_0^1 \eta \Phi_n^2 d\eta} . \tag{30}$$

After substituting Θ and Φ from equations (17) and performing the integration, one obtains

$$A_n = \frac{\frac{c_p}{a\lambda_s}\left[\frac{L^{\frac{1}{2}}J_o(\beta_n L^{\frac{1}{2}})J_1(\beta_n) - J_o(\beta_n)J_1(\beta_n L^{\frac{1}{2}})}{\beta_n L^{\frac{1}{2}} J_o(\beta_n L^{\frac{1}{2}})}\right]}{\frac{c_p}{a\lambda_s}\,\frac{1}{2}[J_o^2(\beta_n) + J_1^2(\beta_n)] + \frac{J_o^2(\beta_n)}{J_o^2(\beta_n L^{\frac{1}{2}})}\,\frac{1}{2}[J_o^2(\beta_n L^{\frac{1}{2}}) + J_1^2(\beta_n L^{\frac{1}{2}})]}\,. \tag{31}$$

Upon employing equation (20), A_n can be simplified to the final form

$$A_n = \frac{\left(\frac{c_p}{a\lambda_s}\right)^2 \left[\frac{J_1(\beta_n)}{\beta_n}\left(\frac{a\lambda_s}{c_p}+1\right)\right]}{\frac{c_p}{a\lambda_s}\,\frac{1}{2}[J_o^2(\beta_n) + J_1^2(\beta_n)] + \frac{J_o^2(\beta_n)}{J_o^2(\beta_n L^{\frac{1}{2}})}\,\frac{1}{2}[J_o^2(\beta_n L^{\frac{1}{2}}) + J_1^2(\beta_n L^{\frac{1}{2}})]} \tag{32}$$

Once A_n has been evaluated, then B_n follows from

$$B_n = -\left(\frac{a\lambda_s}{c_p}\right)\frac{J_o(\beta_n)}{J_o(\beta_n L^{\frac{1}{2}})}\,A_n\,. \tag{33}$$

With the A_n and B_n known, the final forms of the temperature and mass fraction solutions are expressible as

$$\frac{T - T_f}{T_o - T_f} = \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 \xi} J_o(\beta_n \eta) \tag{34}$$

and

$$\frac{C - C_f}{C_o - C_f} = \sum_{n=1}^{\infty} B_n e^{-\beta_n^2 \xi} J_o(\beta_n L^{\frac{1}{2}} \eta)\,. \tag{35}$$

2-3. Application of Solutions

Numerical evaluation of equations (34) and (35) provides the temperature and vapor mass fraction at any point in the tube. The quantities T_f and C_f are determined by solving equation (6) in conjunction with a saturation state in the fully developed region.

The variations of the wall temperature and vapor mass fraction along the length of the tube can be calculated from equations (34) and (35) by introducing $\eta = 1$, namely,

$$\frac{T_w - T_f}{T_o - T_f} = \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 \xi} J_o(\beta_n) , \tag{36}$$

$$\frac{C_w - C_f}{C_o - C_f} = \sum_{n=1}^{\infty} B_n e^{-\beta_n^2 \xi} J_o(\beta_n L^{\frac{1}{2}}) . \tag{37}$$

The bulk temperature variation is determined from its definition

$$T_b = \frac{\int_0^{r_o} 2\pi r u T dr}{\int_0^{r_o} 2\pi r u dr} . \tag{38}$$

Upon substituting T from equation (34) and performing the integration, there is obtained

$$\frac{T_b - T_f}{T_o - T_f} = 2 \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 \xi} \frac{J_1(\beta_n)}{\beta_n} . \tag{39}$$

In a similar manner, by defining a bulk mass fraction as

$$C_b = \frac{\int_0^{r_o} 2\pi r u C dr}{\int_0^{r_o} 2\pi r u dr} \tag{40}$$

and using the series solution (35), one finds

$$\frac{C_b - C_f}{C_o - C_f} = 2 \sum_{n=1}^{\infty} B_n e^{-\beta_n^2 \xi} \frac{J_1(\beta_n L^{\frac{1}{2}})}{\beta_n L^{\frac{1}{2}}} . \tag{41}$$

By comparing the right-hand sides of equations (39) and (41) in light of the second of equations (19), it is seen that

$$\frac{T_b - T_f}{T_o - T_f} = \frac{C_b - C_f}{C_o - C_f}. \tag{42}$$

Other quantities of interest include the local heat and mass flux rates at the wall. By applying Fourier's law, $q = -k\frac{\partial T}{\partial r}$ at $r = r_o$, in conjunction with the temperature solution (34), one obtains

$$\frac{q_w r_o}{k(T_o - T_f)} = \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 \xi} \beta_n J_1(\beta_n). \tag{43}$$

The local rate of mass transfer at the wall is found by employing Fick's law $\dot{m} = \rho D(\frac{\partial C}{\partial r})_{r_o}$ together with equation (35). This yields

$$\frac{\dot{m} r_o}{\rho DL(C_f - C_o)} = \sum_{n=1}^{\infty} B_n e^{-\beta_n^2 \xi} \beta_n \frac{J_1(\beta_n L^{\frac{1}{2}})}{L^{\frac{1}{2}}}. \tag{44}$$

By employing the second of equations (19), it can be readily shown that the left-hand sides of equations (43) and (44) are identical. Thus,

$$\frac{\dot{m} r_o}{\rho DL(C_f - C_o)} = \frac{q_w r_o}{k(T_o - T_f)}. \tag{45}$$

A local Nusselt number Nu_{r_o} based on the radius of the tube can be evaluated in terms of quantities that have already been discussed. Thus,

$$Nu_{r_o} = \frac{q_w}{T_b - T_w} \frac{r_o}{k} = \frac{q_w r_o}{k(T_o - T_f)} \Big/ \left(\frac{T_b - T_f}{T_o - T_f} - \frac{T_w - T_f}{T_o - T_f}\right) \tag{46}$$

or, in series form,

$$Nu_{r_o} = \frac{\sum_{n=1}^{\infty} A_n e^{-\beta_n^2 \xi} \beta_n J_1(\beta_n)}{2 \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 \xi} \frac{J_1(\beta_n)}{\beta_n} - \sum_{n=1}^{\infty} A_n e^{-\beta_n^2 \xi} J_o(\beta_n)}. \qquad (46a)$$

At large values of x, the Nusselt number approaches a fully developed value, the expression for which is deduced from equation (46a) by introducing the leading term (n=1) of each of the constituent series. This gives

$$(Nu_{r_o})_f = \frac{\beta_1^2}{2 - \beta_1 \frac{J_o(\beta_1)}{J_1(\beta_1)}}. \qquad (47)$$

CHAPTER 3

NUMERICAL RESULTS AND DISCUSSION

As was presented in the analysis section, determination of the eigenvalues β_n is an essential feature for the numerical evaluation of the results. The eigenvalues and, consequently, all of the results of engineering interest depend parametrically on the two property groups L and $a\lambda_s/c_p$. In the present investigation, the Lewis number L was assigned values of 0.81, 2.0 and 3.5. This choice was suggested by surveying a wide range of gases and vapors in dilute binary solution with air (8). With respect to the parameter $a\lambda_s/c_p$, values of 0.1, 1, and 10 were selected on the basis of actual property evaluations for air-water vapor and air-naphthalene mixtures at various temperature and pressure levels. Thus, a total of nine cases were treated in the numerical calculations. The computer program used in obtaining the numerical results is given in Appendix C.

3-1. Eigenvalues

The eigenvalues correspond to the roots of equation (20). For each one of the nine cases 55 eigenvalues were determined. They are listed in Table B-1 of Appendix B. In the actual computations, 40 eigenvalues were found to be sufficient for providing numerical results of high accuracy for the cases of L = 2.0 and 3.5, while 50 eigenvalues were necessary for the case with L = 0.81.

Once the eigenvalues are available, the coefficients A_n

and B_n may be evaluated from equations (32) and (33). In turn, once the A_n and B_n are known, all results of engineering interest can be calculated from the equations derived in the analysis section.

3-2. Bulk Temperature and Mass Fraction

The axial variations of bulk temperature and mass fraction are listed in Table B-2, Appendix B. They are also presented graphically in Figure 3-1. In terms of the dimensionless ordinate variables, the axial distributions of bulk temperature and bulk mass fraction are identical (see equation (42)). The abscissa variable represents the dimensionless axial distance from the entrance, the denominator Ur_o/D being the Peclet number for diffusion. In constructing the figure, it was found that the use of $(x/r_o)/(Ur_o/D)$ gives a more compact presentation than does the ξ variable. The curves are plotted as solid, dot-dashed and dashed lines, respectively for Lewis numbers of 0.81, 2.0, and 3.5. For each Lewis number, there are curves for $a\lambda_s/c_p$ = 0.1, 1, and 10. To preserve clarity, the results for L = 0.81 and 3.5 are referred to the lower abscissa, while those for L = 2.0 are referred to the upper abscissa.

It is evident from the figure that all curves decrease as the downstream distance increases. Since $T_o > T_f$, it follows that T_b decreases along the tube. On the other hand, $C_o < C_f$, and C_b increase with increasing downstream distance. These trends are consistant with physical reasoning. The curves decrease very rapidly at small values of x, but the

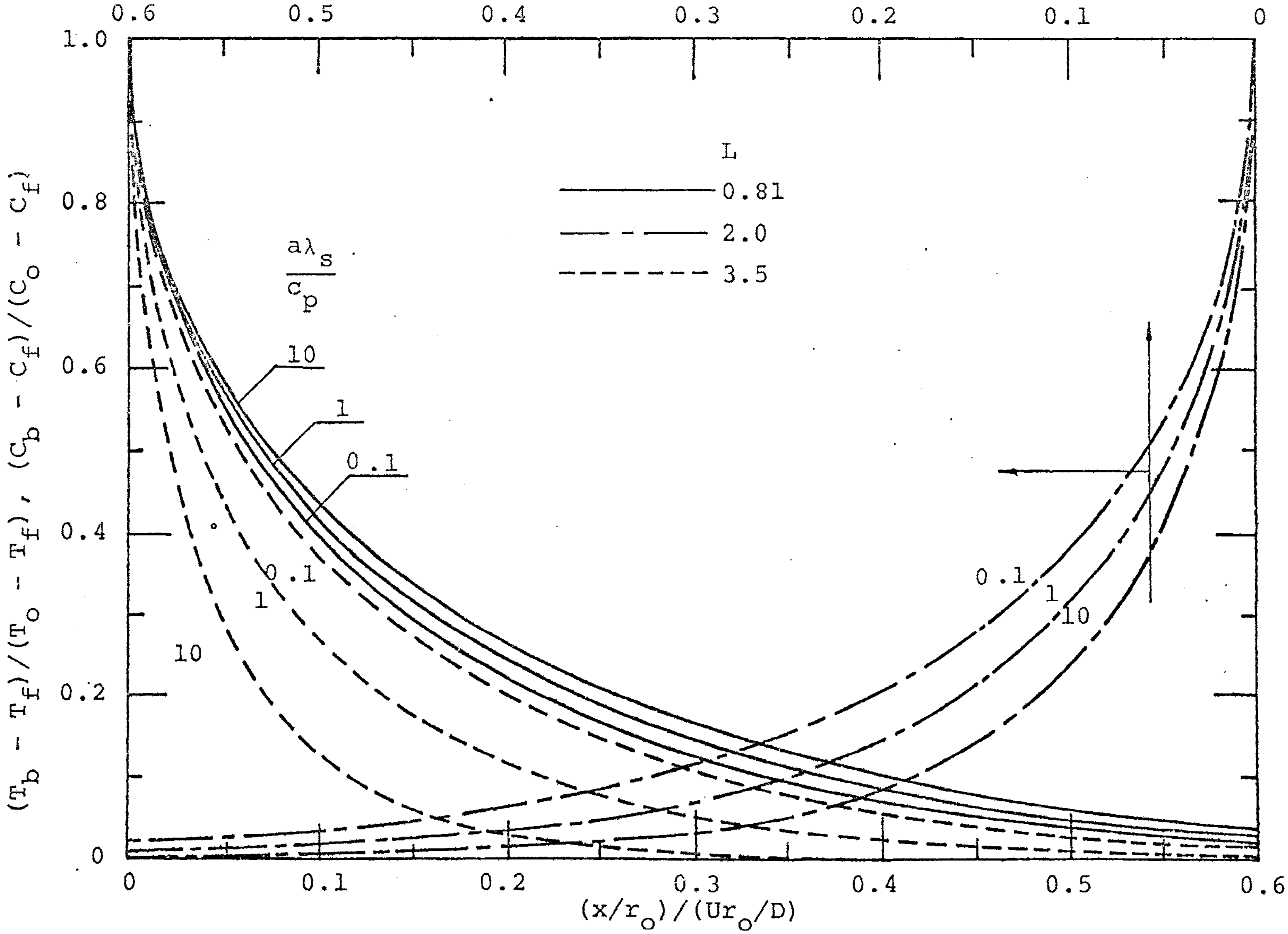

Figure 3-1. Bulk temperature and mass fraction distributions.

rates of change become increasingly slower as x becomes larger. These results indicate the fact that the local heat and mass fluxes are very high in the neighborhood of the tube inlet and decrease with increasing downstream distance.

The rate of sublimation from the wall for any axial length of tube can be computed by employing the bulk mass fraction results. It can be easily demonstrated that by integrating the second of equations (3) across the section and along the length of the tube, the sublimation rate between $x = 0$ and any axial station x is given by $\dot{M}(C_b - C_o)$, where $\dot{M}$ is the axial rate of mass flow and C_b is the bulk mass fraction at x. It then follows that the sublimation rate in a length of tube between two sections x_1 and x_2 is $\dot{M}(C_{b2} - C_{b1})$, wherein C_{b2} and C_{b1} are, respectively, the bulk mass fractions at x_2 and x_1.

Further inspection of Figure 3-1 reveals that the dimensionless bulk temperature and mass fraction distributions are relatively insensitive to $a\lambda_s/c_p$ when $L = 0.81$. As L increases, the variations become increasingly sensitive to $a\lambda_s/c_p$. Moreover, the curves with $a\lambda_s/c_p$ are arranged in ascending order when $L < 1$, and in descending order when $L > 1$. For $L < 1$, it appears that the transport processes are more rapid with decreasing $a\lambda_s/c_p$; the opposite trend is in evidence for $L > 1$. These remarks about the ordering of the curves with $a\lambda_s/c_p$ and relative development rates are relevant only for common values of Ur_o/D.

3-3. Entrance Lengths

The values of the bulk temperature and mass fraction characterizing the near approach to fully developed conditions are employed to determine the entrance length. In this investigation, the entrance length is defined as the condition corresponding to

$$\frac{T_b - T_f}{T_o - T_f} = \frac{C_b - C_f}{C_o - C_f} = 0.05 \,. \tag{48}$$

The results are obtained from equation (46a) by taking its leading term (n = 1) and are listed in column 3 of Table 3-1 in terms of the quantity $(x/r_o)/(Ur_o/D)$. For purposes of comparison, the entrance lengths for slug flow in a parallel-plate channel (4) are listed in column 4 of the table.

From the table, it is seen that the dimensionless entrance lengths characterized by the parameter $(x/r_o)/(Ur_o/D)$ range from about 0.15 to 0.55, with a tendency toward higher values at lower Lewis numbers (for a fixed $a\lambda_s/c_p$). When the diameter d of the tube is used as the characteristic dimension instead of the radius, the dimensionless entrance lengths $(x/d)/(Ud/D)$ range from about 0.04 to 0.14. Entrance lengths for laminar heat transfer and mass transfer in tubes typically lie in this range (1).

Inspection of Table 3-1 reveals that the entrance length has a relatively weak dependence on $a\lambda_s/c_p$ when L = 0.81, decreasing with decreasing values of this parameter. At larger L, this trend is reversed and a stronger dependence is also in evidence. Also, the dimensionless entrance length

TABLE 3-1.

ENTRANCE LENGTHS

(Slug flow profile)

L	$\frac{a\lambda_s}{c_p}$	Circular Tube $\frac{x/r_o}{Ur_o/D}$	Parallel-plate Channel $\frac{x/b}{Ub/D}$
0.81	0.1	0.464	1.154
0.81	1.0	0.503	1.263
0.81	10.0	0.551	1.370
2.0	0.1	0.435	1.081
2.0	1.0	0.346	0.858
2.0	10.0	0.246	0.620
3.5	0.1	0.428	1.061
3.5	1.0	0.306	0.751
3.5	10.0	0.145	0.406

is very insensitive to L for small values of $a\lambda_s/c_p$, but is quite sensitive to L when $a\lambda_s/c_p$ becomes larger.

The entrance lengths for channel flow, based on the hydraulic diameter D_h, are about 62 percent of those for tube flow. Closer inspection shows that these two sets of results have a remarkably constant relationship over the entire range of parameters investigated.

3-4. Wall Temperature and Mass Fraction

The longitudinal distributions of the wall temperature are listed in Table B-3 of Appendix B. They are plotted in Figure 3-2, with $(x/r_o)/(Ur_o/D)$ as the abscissa. The curves are parameterized by L and $a\lambda_s/c_p$ in a manner similar to that of Figure 3-1. In view of equation (10), that is

$$\frac{C_w - C_f}{C_o - C_f} = -\left(\frac{a\lambda_s}{c_p}\right)\left(\frac{T_w - T_f}{T_o - T_f}\right), \tag{49}$$

there is no need to make a separate presentation of the results for the wall mass fraction.

Inspection of the figure reveals two different trends. When $L < 1$, the wall temperature T_w is always lower than its fully developed value T_f, taking on its minimum value at $x = 0$ and increasing with increasing axial distance. On the other hand, when $L > 1$, T_w always exceeds T_f, assuming its maximum value at $x = 0$ and decreasing steadily as x increases.

The wall temperature at $x = 0$ tends to approach more closely to T_o as $a\lambda_s/c_p$ decreases when $L > 1$; correspondingly the curves are arranged in ascending order in accordance with

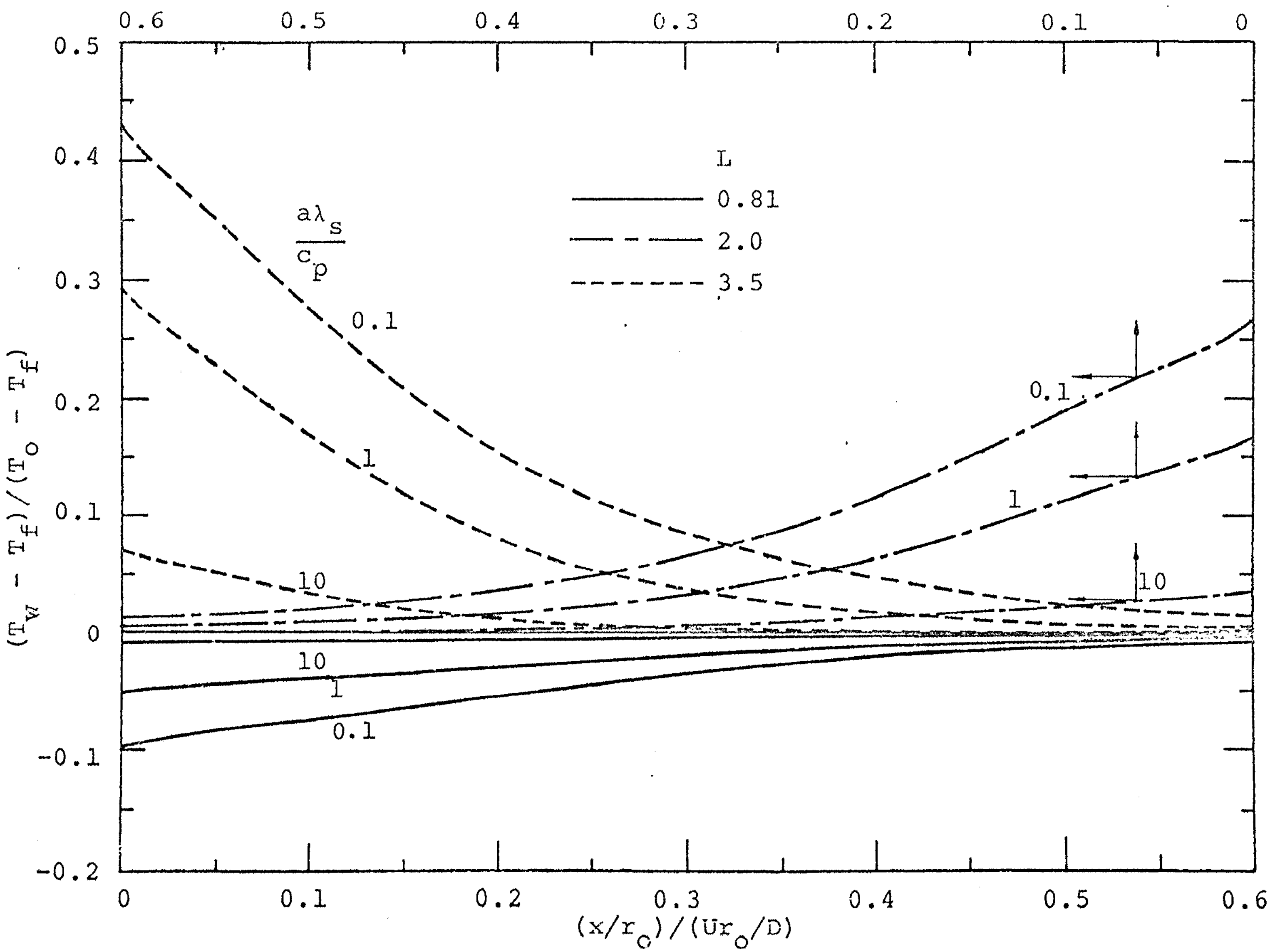

Figure 3-2. Wall temperature distributions.

TABLE 3-2.

WALL TEMPERATURE RESULTS $(T_w - T_f)/(T_o - T_f)$ NEAR TUBE INLET

(Slug flow profile)

L	$\frac{a\lambda_s}{c_p}$	Circular Tube*	Parallel-plate Channel**	Leveque-type solution (4)
0.81	0.1	-0.097302	-0.099997	-0.100000
0.81	1.0	-0.051187	-0.052631	-0.052631
0.81	10.0	-0.008925	-0.009173	-0.009174
2.0	0.1	0.267949	0.273544	0.273549
2.0	1.0	0.167821	0.171573	0.171574
2.0	10.0	0.035378	0.036289	0.036289
3.5	0.1	0.434862	0.441856	0.441860
3.5	1.0	0.297781	0.303334	0.303338
3.5	10.0	0.071592	0.073358	0.073359

*$\xi = (x/r_o)/(Ur_o/\alpha) = 0.002$; **$\xi = (x/b)/(Ub/\alpha) = 0.005$ (4)

decreasing $a\lambda_s/c_p$. Opposite trends are in evidence when $L < 1$.

It is of interest to compare the wall temperature results for small values of axial distance with those for the case of slug flow in a parallel-plate channel. For this purpose, Table 3-2 has been prepared. In column 3 are listed the results from the series solution for the case of tube flow at $(x/r_o)/(Ur_o/\alpha) = 0.002$. Columns 4 and 5 show the results respectively from the series solution for channel flow at $(x/b)/(Ub/\alpha) = 0.005$ and the Leveque-type (i.e. boundary layer) solution. These two sets of results were taken from reference 4.

Inspection of the table reveals remarkably good agreement among the three sets of results for all of the nine cases considered. The results from the tube solution are only a few percent lower than those from both the channel and the boundary layer solutions. This is to be expected because in the immediate neighborhood of the tube inlet, the boundary layer thickness is very thin compared to the radius of the tube; hence, the effect of curvature can be neglected. The fact that the results from the series solution for tube flow are in excellent agreement with those from the Leveque-type solution lends strong support to the computational validity of the series solution.

3-5. Wall Heat and Mass Fluxes

The numerical results for the wall heat and mass fluxes are listed in Table B-4, Appendix B. A graphical presentation

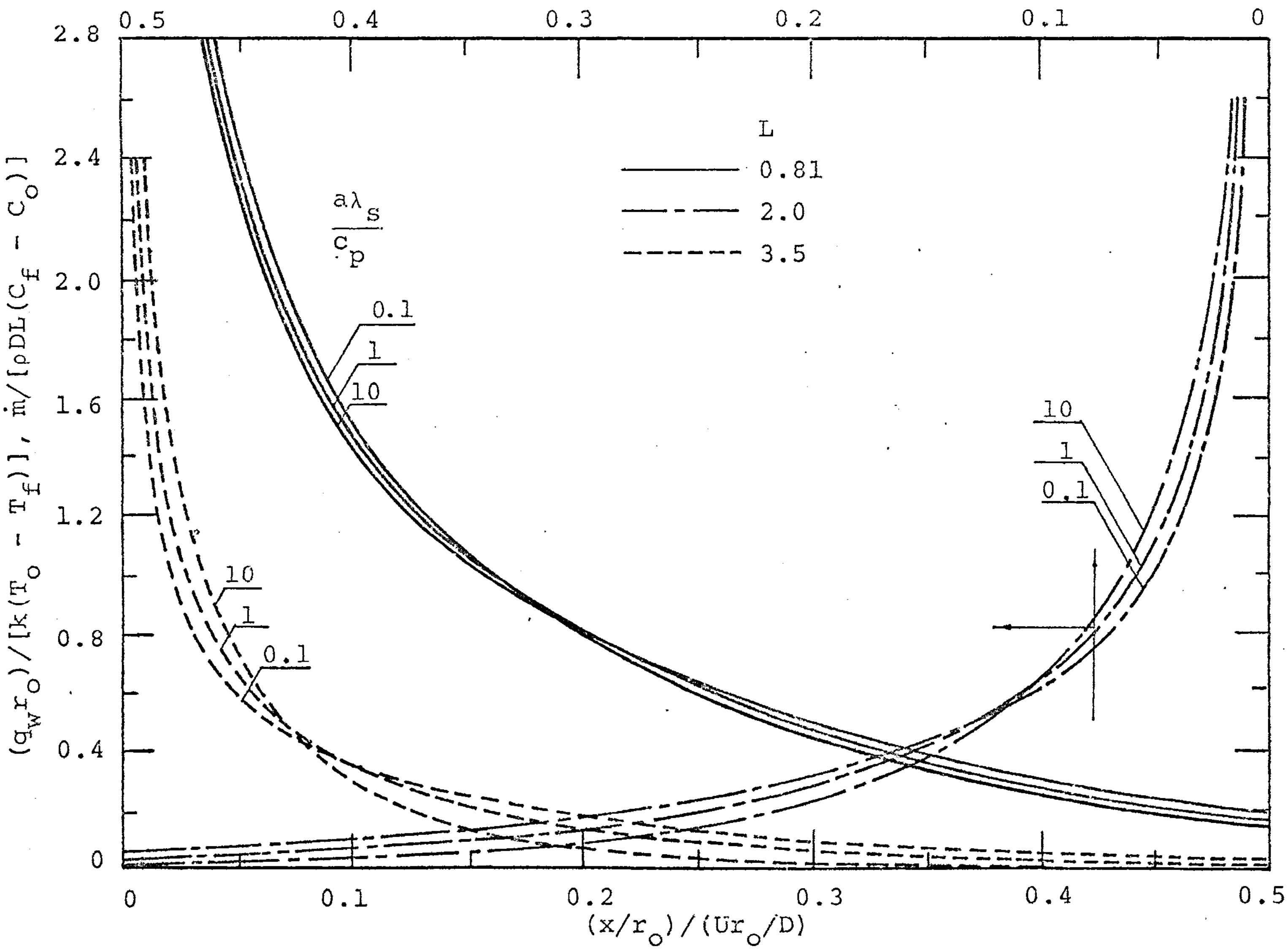

Figure 3-3. Wall heat and mass flux distributions.

of the variations of these groups with distance along the tube is made in dimensionless form in Figure 3-3. The structure of the figure and parameterization of the curves are similar to that of Figure 3-1.

It is evident from the figure that the local wall heat and mass transfer rates take on very high values in the region immediately adjacent to the tube inlet and decrease monotonically with increasing axial distance, approaching zero asymptotically.

The dimensionless heat transfer and mass transfer parameters are seen to be relatively insensitive to $a\lambda_s/c_p$; in particular, for L = 0.81. This does not necessarily imply that the wall heat flux or the wall mass flux has a weak dependence on $\dot{a}\lambda_s/c_p$, since the dimensionless transfer parameters contain the quantities $(T_o - T_f)$ and $(C_o - C_f)$, which are not independently prescribable, but are dependent on $a\lambda_s/c_p$ through equations (8) and (6). Furthermore, the curves are ordered so that smaller values of L are associated with higher values of the dimensionless transfer parameters. Again, this may not necessarily correspond to a similar ordering for q_w and $\dot{m}$.

3-6. The Nusselt Numbers

The local Nusselt number, defined in equation (46) or (46a), provides a direct measure of the local heat transfer coefficient. The local Nusselt number results are listed in Table B-5, Appendix B. They are also illustrated in Figure 3-4.

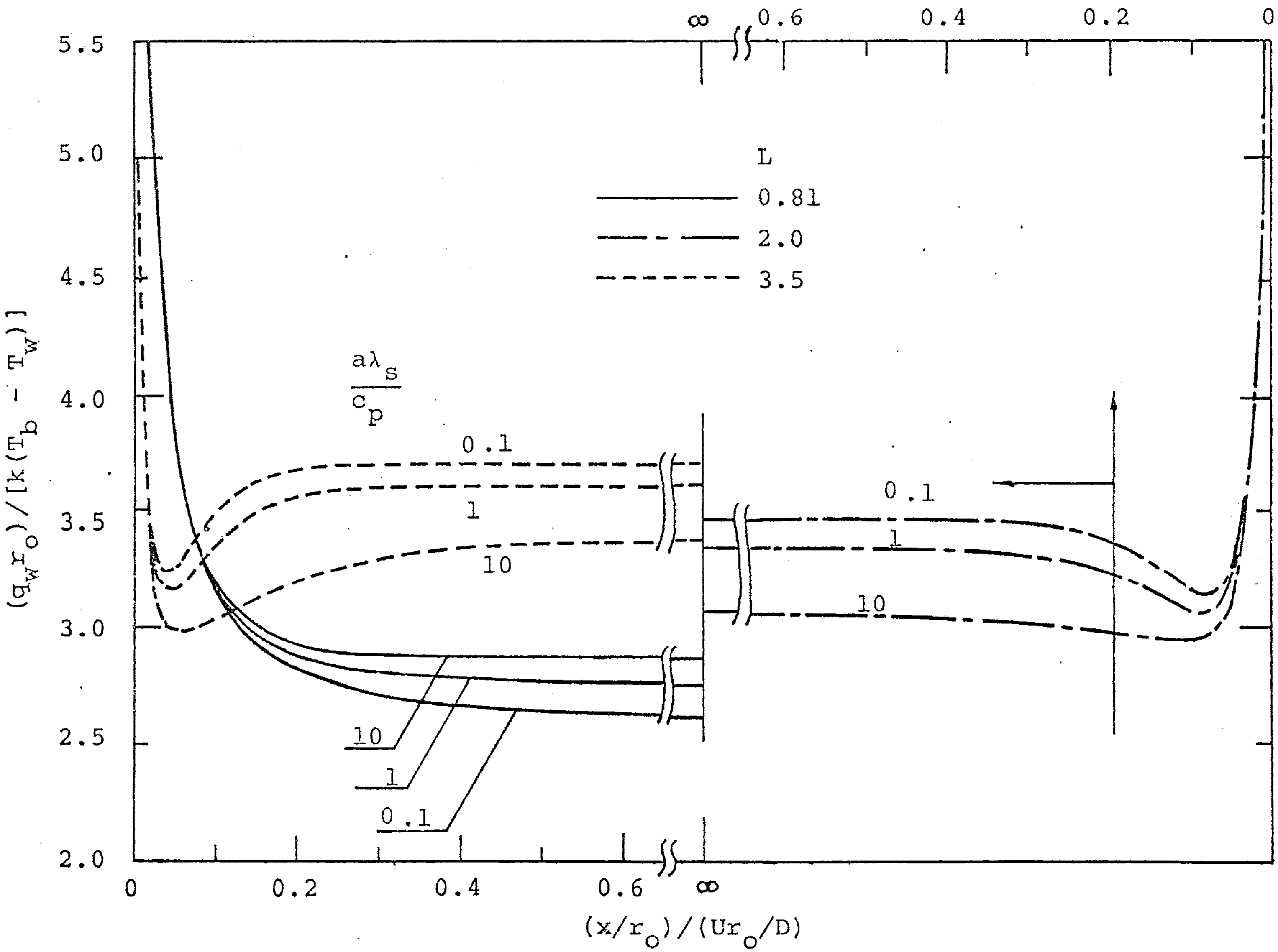

Figure 3-4. Nusselt number distributions.

It is seen from the figure that, aside from the immediate neighborhood of the entrance section, where very high values are attained, the local Nusselt numbers for all cases vary only slightly with increasing downstream distance. This behavior shows quite a contrast to Figures 3-1, 3-2, and 3-3, where in most cases, T_b, T_w, and q_w display appreciable axial variation in the same region in which the Nusselt number is relatively unchanging.

Further inspection of Figure 3-4 reveals that the axial variation of the Nusselt number is somewhat unusual. It does not follow the pattern normally encountered in the duct flow heat transfer problem with conventional boundary conditions such as uniform wall temperature or uniform wall heat flux. For these thermal conditions, the Nusselt number decreases monotonically, approaching the respective fully developed limits from above. On the other hand, Figure 3-4 shows that for the cases of L = 2.0 and 3.5 the Nusselt numbers attain minimum values somewhere in the close vicinity of the tube inlet and then approach the fully developed limits from below.

It is of interest to compare the fully developed Nusselt numbers with those for the limiting cases of uniform wall temperature and uniform heat flux. The values of Nu_{r_o} for these limiting cases are 2.892 and 4, respectively. They are calculated in Appendix A. For the present problem, the results are listed in Table 3-3, where, for the purpose of comparison, the results for the case of slug flow in a parallel-plate channel are also given.

It is seen from the table that the fully developed Nusselt numbers for $L = 0.81$ are lower than those for both uniform wall temperature and uniform wall heat flux (i.e., less than 2.892), while those for $L > 1$ are bracketed between the values for uniform wall temperature and uniform wall heat flux (i.e., between 2.892 and 4). However, Figure 3-4 has shown that the Nu_{r_o} distributions for $L > 1$ display upstream minima, suggesting that in some portion of the entrance region, these Nusselt numbers fall lower than that for the uniform wall temperature. These findings confirm that the Nusselt number results for the standard cases of uniform wall temperature and uniform wall heat flux are not necessarily universal bounds.

The fully developed Nusselt number results for the present problem are compared with those for the case of channel flow, respectively in columns 3 and 4 of Table 3-3. It is seen from the table that the two sets of results show a similar pattern of dependence on Lewis number L and on the property group $a\lambda_s/c_p$, the values for channel flow being smaller when based on the half-height of the channel. For channel flow, the fully developed Nusselt numbers Nu_b for the limiting cases of uniform wall temperature and uniform wall heat flux are found to be 2.467 and 3, respectively. Inspection of column 4 in the table reveals that the Nusselt numbers for channel flow are lower than 2.467 and 3 when $L = 0.81$ and are bracketed between the values 2.467 and 3

TABLE 3-3

FULLY DEVELOPED NUSSELT NUMBERS

(Slug flow profile)

L	$\frac{a\lambda_s}{c_p}$	Circular Tube $Nu_{r_o} = \frac{hr_o}{k}$	Parallel-plate channel $Nu_b = \frac{hb}{k}$
0.81	0.1	2.616	2.346
0.81	1.0	2.762	2.408
0.81	10.0	2.871	2.458
2.0	0.1	3.465	2.733
2.0	1.0	3.336	2.664
2.0	10.0	3.059	2.526
3.5	0.1	3.699	2.847
3.5	1.0	3.601	2.789
3.5	10.0	3.369	2.626

when L = 2.0 and L = 3.5, exhibiting an identical pattern displayed by the case of tube flow. These observations reaffirm the fact that the limiting cases of uniform wall temperature and uniform heat flux are not universal bounds.

3-7. Temperature and Mass Fraction Profiles

The numerical results of temperature and mass fraction distributions are listed, respectively, in Table B-6 and Table B-7 of Appendix B. In order to preserve clarity, only representative profiles are shown in Figures 3-5, 3-6, 3-7, and 3-8. Consideration is first given to the temperature profiles, Figures 3-5 and 3-6. Each of these figures shows a sequence of temperature profiles for various dimensionless axial distances ξ. Results for $a\lambda_s/c_p$ = 0.1 and 10 are respectively denoted by dashed and solid lines.

Inspection of the figures reveals that the temperature field is of the boundary layer type in the immediate neighborhood of the tube inlet. That is, the temperature gradients are confined to a thin layer near the wall and there is an isothermal core. With increasing axial distance, the effect of the wall heat transfer extends into the bulk of the fluid, causing the temperature to decrease. In general, the temperature profiles become flatter and flatter as the axial distance increases, ultimately approaching the fully developed, uniform value T_f across the entire tube cross section.

For L = 0.81, Figure 3-5, the temperature profiles do not depend strongly on the property group $a\lambda_s/c_p$. On the other hand, for L = 3.5, Figure 3-6, the profiles are

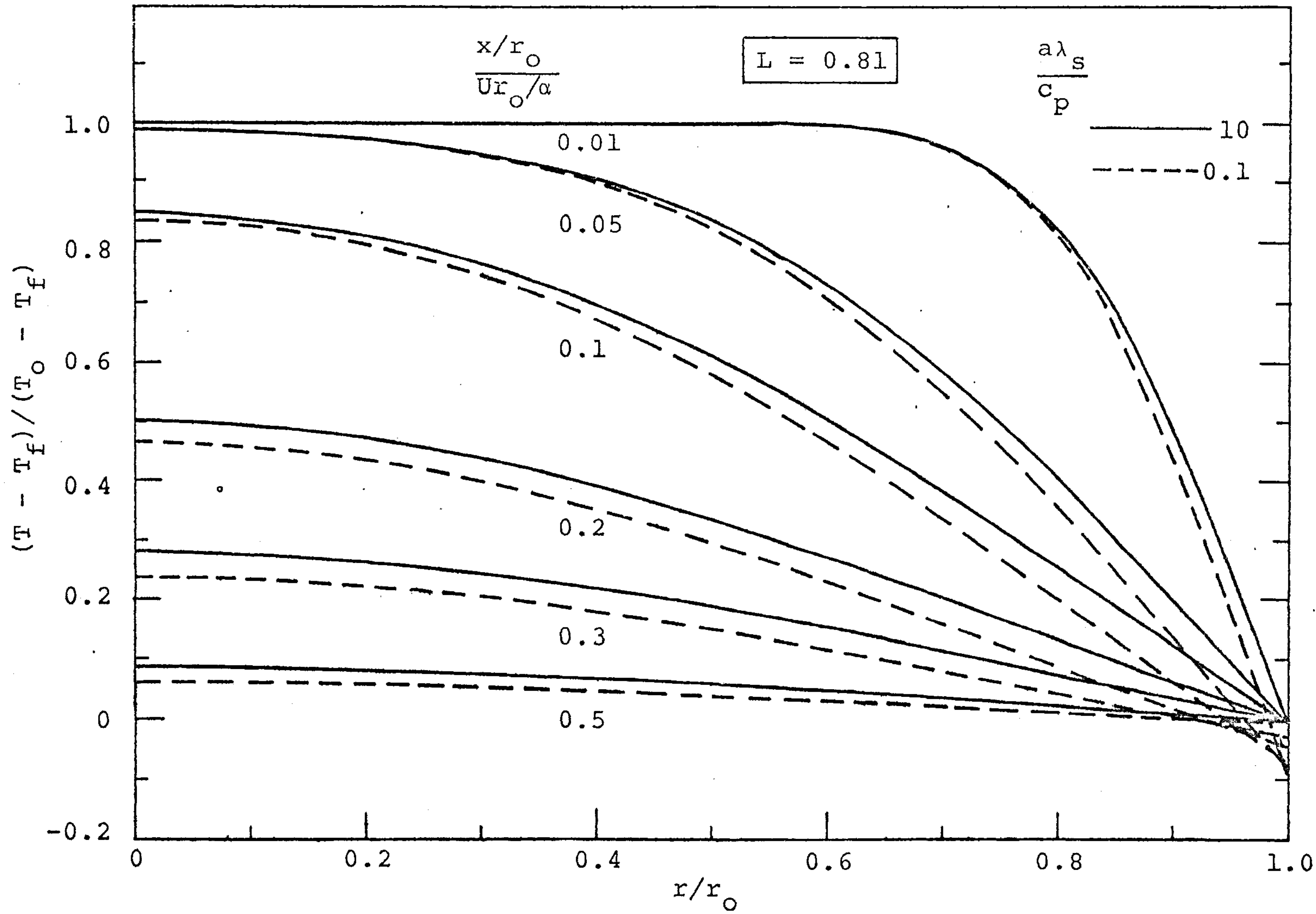

Figure 3-5. Temperature profiles, L = 0.81.

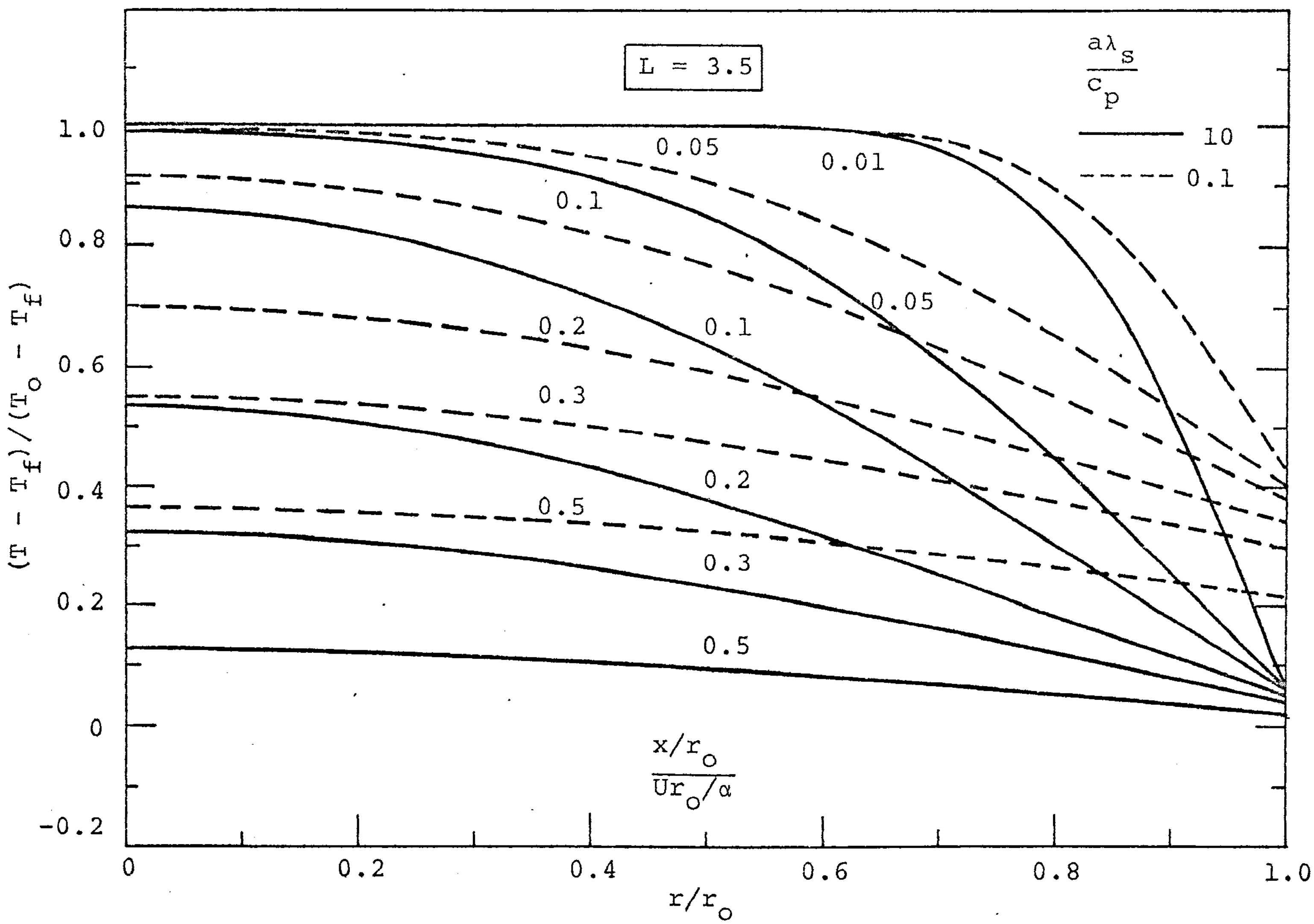

Figure 3-6. Temperature profiles, L = 3.5.

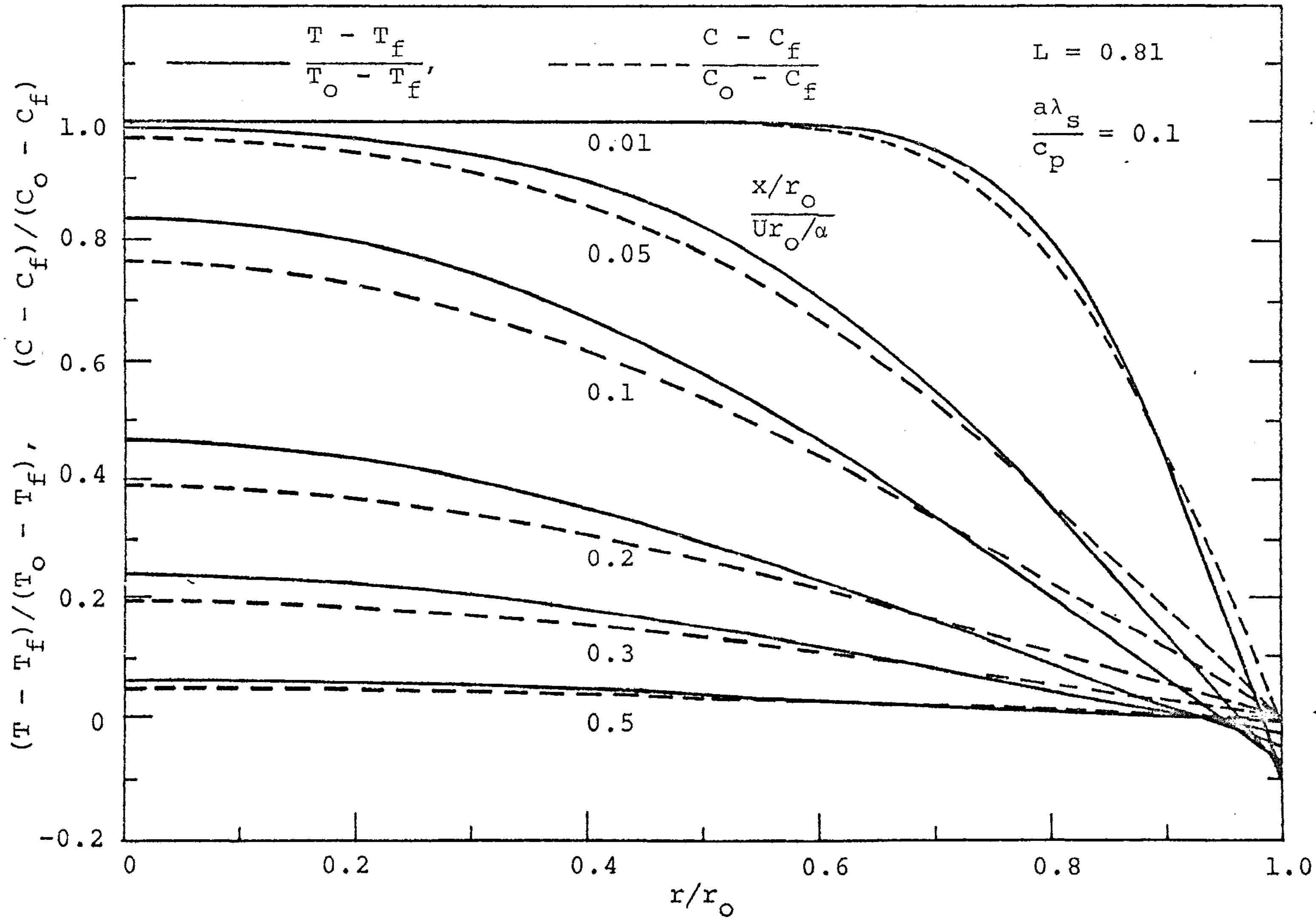

Figure 3-7. Comparison between temperature and mass fraction profiles, L = 0.81.

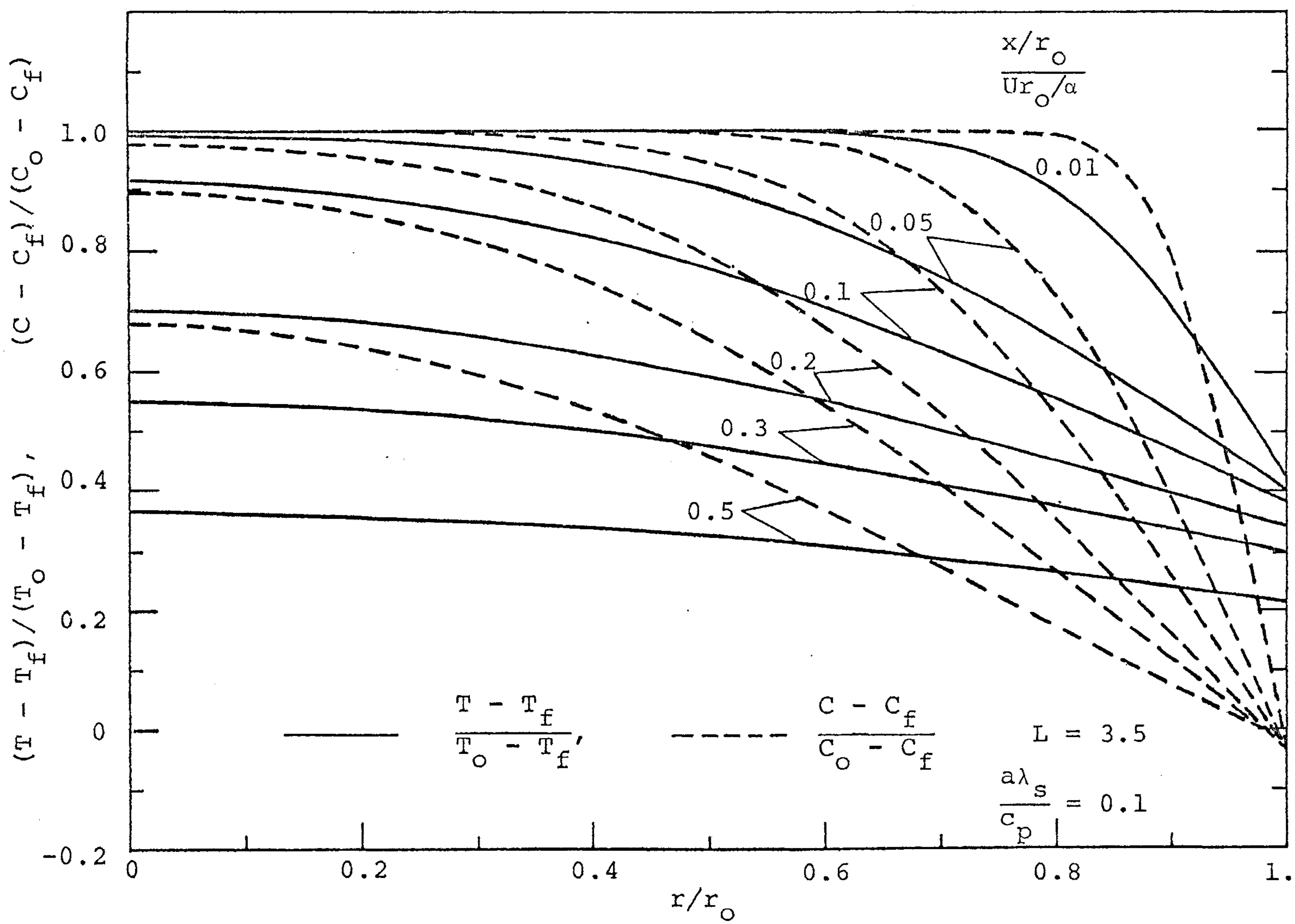

Figure 3-8. Comparison between temperature and mass fraction profiles, L = 3.5.

markedly different for different values of $a\lambda_s/c_p$, with those for smaller $a\lambda_s/c_p$ being flatter.

Next, attention is given to comparisons between the mass fraction and temperature profiles, Figure 3-7 and Figure 3-8. These figures correspond, respectively, to L = 0.81 and L = 3.5. All of the curves in these figures are for $a\lambda_s/c_p = 0.1$. It is seen that for L = 0.81, the shapes of the corresponding mass fraction and temperature profiles are quite similar. This similarity is to be expected, since the heat and mass transfer processes are nearly analogous for Lewis numbers close to unity. On the other hand, marked differences exist between the temperature and mass fraction profiles when L = 3.5, marking a noticeable departure from the analogy between heat and mass transfer.

CHAPTER 4

CONCLUDING REMARKS

The combined heat and sublimation mass transfer in laminar tube flow was investigated. The analysis of the problem required the simultaneous solution of the energy and diffusion equations, with coupling provided by the boundary conditions. An analytical solution of the problem was facilitated by employing a linear saturation state relationship for the vapor of the subliming solid. The eigenvalue problem encountered in this study did not fall within the conventional Sturm-Liouville family and a special integral relation was developed which served as an orthogonality condition. Mathematical expressions were derived and numerical results obtained for various quantities of engineering interest.

From the problem investigated, the following conclusions can be drawn:

(1) The eigenvalues depend on two parameters, the Lewis number L and the property group $a\lambda_s/c_p$.

(2) The bulk temperature decreases with increasing axial distance, while the bulk mass fraction increases.

(3) The wall temperature decreases with increasing axial distance when $L > 1$, and increases when $L < 1$.

(4) The wall heat and mass fluxes decrease as the axial distance increases.

(5) In general, the Nusselt numbers decrease as the axial distance increases. However, for the cases with Lewis

numbers larger than 1, the Nusselt numbers attain minimum values somewhere in the entrance region of the tube and then approach the fully developed values from below. This is in contrast to the behavior displayed by duct flow heat transfer problems with conventional thermal boundary conditions, wherein the Nusselt numbers decrease monotonically and approach the fully developed values from above.

(6) The temperature and mass fraction profiles are of the boundary layer type in the immediate neighborhood of the tube inlet. As the axial distance increases, these profiles become flatter and flatter, finally approaching the uniform, fully developed values.

(7) The heat and mass transfer processes are nearly analogous when Lewis number is close to unity. As the Lewis number departs from unity, marked differences exist between the analogy.

(8) The entrance lengths characterized by the near approach to the fully developed values for the bulk temperature and mass fraction fall within those of the conventional problems of pure heat transfer or pure mass transfer.

(9) The wall temperature values in the immediate vicinity of the tube inlet agree with those of the Leveque-type solution. This lends strong support to the computational validity of the series solution.

CHAPTER 5

RECOMMENDATIONS

The following recommendations are made for further study in connection with coupled transport of heat and mass in laminar tube flows.

(1) In this investigation, a slug flow profile is assumed. The analytical model and solution method employed may be extended to a situation in which the parabolic velocity profile prevails. In this instance, the mathematical system can be solved only by numerical methods.

(2) Instead of the linearized saturation state used in the present investigation, other saturation state relationships for the solid-vapor system may be employed.

NOMENCLATURE

A,B	series coefficients, eqs. (32) and (33)
a	constant, eq. (8)
b	constant, eq. (8); half-height of channel
C	mass fraction of vapor
c_p	specific heat at constant pressure
D	binary diffusion coefficient
d	diameter of tube, $2r_o$
h	heat transfer coefficient, $q_w/(T_b - T_w)$
k	thermal conductivity
L	Lewis number, α/D
$\dot{M}$	axial flow rate
$\dot{m}$	local sublimation rate/area
Nu_{r_o}	Nusselt number, hr_o/k
Pe	Peclet number, $(Pr)(Re_d)$
Pr	Prandtl number, ν/α
q	local heat flux/time-area
Re_d	Reynolds number, $\rho Ud/\mu$
r	radial coordinate
r_o	radius of tube
T	temperature
U	average velocity
u	axial velocity
x	axial coordinate
α	thermal diffusivity, $k/\rho c_p$
β	eigenvalue

NOMENCLATURE

A,B	series coefficients, eqs. (32) and (33)
a	constant, eq. (8)
b	constant, eq. (8); half-height of channel
C	mass fraction of vapor
c_p	specific heat at constant pressure
D	binary diffusion coefficient
d	diameter of tube, $2r_o$
h	heat transfer coefficient, $q_w/(T_b - T_w)$
k	thermal conductivity
L	Lewis number, α/D
$\dot{M}$	axial flow rate
$\dot{m}$	local sublimation rate/area
Nu_{r_o}	Nusselt number, hr_o/k
Pe	Peclet number, (Pr)(Re_d)
Pr	Prandtl number, ν/α
q	local heat flux/time-area
Re_d	Reynolds number, $\rho Ud/\mu$
r	radial coordinate
r_o	radius of tube
T	temperature
U	average velocity
u	axial velocity
x	axial coordinate
α	thermal diffusivity, $k/\rho c_p$
β	eigenvalue

η	dimensionless coordinate, r/r_o
Θ	function of η
θ	dimensionless temperature, $(T - T_f)/(T_o - T_f)$; $(T - T_w)/(T_o - T_w)$; $(T - T_w)/(qr_o/k)$
λ_s	latent heat of sublimation
μ	dynamic viscosity
ν	kinematic viscosity, μ/ρ
ξ	dimensionless coordinate, $(x/r_o)/(Ur_o/\alpha)$; $(x/r_o)/Pe$
ρ	density
Φ	function of η
ϕ	dimensionless mass fraction, $(C - C_f)/(C_o - C_f)$

Subscripts

b	bulk
f	fully developed condition
o	entrance section
w	wall

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APPENDICES

Appendix A

FULLY DEVELOPED NUSSELT NUMBERS FOR PURE HEAT TRANSFER PROCESSES

In this appendix, the fully developed Nusselt numbers for laminar heat transfer in a circular tube with conventional boundary conditions will be determined. The case of slug flow is considered.

1. Uniform Wall Temperature

The governing energy equation is

$$\rho c_p u \frac{\partial T}{\partial x} = k \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) . \tag{A-1}$$

The boundary conditions are

$$T = T_o \text{ at } x = 0$$

$$T = T_w \text{ at } r = r_o$$

and

$$\frac{\partial T}{\partial r} = 0 \text{ at } r = 0 .$$

Introducing the dimensionless variables

$$\theta = \frac{T - T_w}{T_o - T_w}, \qquad \eta = \frac{r}{r_o}, \qquad \xi = \frac{x/r_o}{Pe} \tag{A-2}$$

one can write equation (A-1) in dimensionless form as

$$\frac{\partial \theta}{\partial \xi} = \frac{2}{\eta} \frac{\partial}{\partial \eta}\left(\eta \frac{\partial \theta}{\partial \eta}\right) \tag{A-3}$$

in which the slug flow condition $u/U = 1$ has been used. The boundary conditions now have the form

$$\eta = 0 \qquad \frac{\partial\theta}{\partial\eta} = 0 \qquad \text{(symmetry)}$$

$$\eta = 1 \qquad \theta = 0 \tag{A-4}$$

$$x = 0 \qquad \theta = 1$$

$$x \to \infty \qquad \theta \to 0 \; .$$

To solve equation (A-3), one employs the separation of variables method. It can be readily shown that the solution of equation (A-3) assumes the form

$$\theta = \sum_{n=1}^{\infty} C_n e^{-\beta_n^2 \xi} \, \Theta_n(\eta) \tag{A-5}$$

where Θ_n obeys

$$\frac{d}{d\eta}\left(\eta \frac{d\Theta_n}{d\eta}\right) + \beta_n^2 \; \frac{\eta}{2} \, \Theta_n = 0 \tag{A-6}$$

and the boundary conditions

$$\eta = 0, \quad \frac{d\Theta_n}{d\eta} = 0; \qquad \eta = 1, \quad \Theta_n(1) = 0 \tag{A-7}$$

wherein β_n and Θ_n are, respectively, the eigenvalues and eigenfunctions and C_n are the series coefficients.

To determine the series coefficients C_n, one applies the inlet condition that $\theta = 1$ at $\xi = 0$. Thus,

$$1 = \sum_{n=1}^{\infty} C_n \Theta_n(\eta) \, . \tag{A-8}$$

By invoking the orthogonality relation for the Sturm-Liouville system, there is obtained

$$C_n = \left(\int_0^1 \Theta_n \eta d\eta\right) / \left(\int_0^1 \Theta_n^2 \; \eta d\eta\right) . \tag{A-9}$$

The wall heat flux is found from Fourier's law $q_w = k(\frac{\partial T}{\partial r})_{r_o}$. With the aid of equation (A-5), one obtains

$$\frac{q_w r_o}{k(T_o - T_w)} = \sum_{n=1}^{\infty} C_n \Theta_n'(1) e^{-\beta_n^2 \xi} . \qquad \text{(A-10)}$$

Next, from the definition of the bulk temperature

$$T_b = \frac{\int_0^{r_o} 2\pi r u T dr}{\int_0^{r_o} 2\pi r u dr} \qquad \text{(A-11)}$$

one finds

$$\frac{T_b - T_w}{T_o - T_w} = 2\int_0^1 \theta \, \eta d\eta . \qquad \text{(A-12)}$$

The right-hand side of equation (A-12) is determined as follows. First, equation (A-6) is rewritten as

$$\frac{d}{d\eta}(\eta \frac{d\Theta_n}{d\eta}) = -\beta_n^2 \frac{\eta}{2} \Theta_n . \qquad \text{(A-13)}$$

Upon integrating equation (A-13) from $\eta = 0$ to $\eta = 1$, one finds

$$\Theta_n'(1) = -\beta_n^2 \int_0^1 \frac{\eta}{2} \Theta_n d\eta . \qquad \text{(A-14)}$$

By employing equation (A-5) and making use of equation (A-14), equation (A-12) can be reduced to

$$\frac{T_b - T_w}{T_o - T_w} = -4 \sum_{n=1}^{\infty} C_n \frac{\Theta_n'(1)}{\beta_n^2} e^{-\beta_n^2 \xi} . \qquad \text{(A-15)}$$

The Nusselt number can be found from the definition

$$Nu_d = \frac{hd}{k} = \frac{q_w}{T_o - T_w}\frac{2r_o}{k}\left(-\frac{T_o - T_w}{T_b - T_w}\right). \qquad \text{(A-16)}$$

Substitution of equations (A-10) and (A-15) into equation (A-16) yields

$$Nu_d = \frac{\sum_{n=1}^{\infty} C_n\, \Theta_n'(1) e^{-\beta_n^2 \xi}}{2\sum_{n=1}^{\infty} C_n \frac{\Theta_n'(1)}{\beta_n^2}\, e^{-\beta_n^2 \xi}}. \qquad \text{(A-17)}$$

For very large values of ξ, the Nusselt number approaches a fully developed value which can be obtained from equation (A-17) by taking only the leading term of each of the series. Thus,

$$(Nu_d)_f = \frac{1}{2}\, \frac{C_1\ \Theta_1'(1) e^{-\beta_1^2 \xi}}{\frac{C_1\ \Theta_1'(1)}{\beta_1^2}\, e^{-\beta_1^2 \xi}} = \frac{1}{2}\beta_1^2. \qquad \text{(A-18)}$$

In order to calculate the fully developed Nusselt number, one needs the first eigenvalue β_1. To this end, one has to solve equation (A-6) subject to the boundary conditions (A-7). This leads to

$$\Theta_n \sim J_o\left(\frac{\beta_n}{\sqrt{2}}\eta\right). \qquad \text{(A-19)}$$

Application of the second of boundary conditions (A-7) yields

$$J_o\left(\frac{\beta_n}{\sqrt{2}}\right) = 0. \qquad \text{(A-20)}$$

The first root of equation (A-20) is

$$\frac{\beta_1}{\sqrt{2}} = 2.405.$$

Thus, equation (A-18) gives the fully developed Nusselt number as

$$Nu_d = \frac{\beta_1^2}{2} = 5.784$$

or in terms of the tube radius r_o,

$$Nu_{r_o} = \frac{hr_o}{k} = 2.892. \tag{A-21}$$

2. <u>Uniform Wall Heat Flux</u>

The fully developed heat transfer condition is characterized by

$$\frac{\partial}{\partial x}\left(\frac{T - T_w}{T_b - T_w}\right) = 0. \tag{A-22}$$

For uniform wall heat flux, the energy balance on a control volume gives

$$\frac{dT_b}{dx} = \frac{2q_w}{\rho u c_p r_o} = \text{const.} \tag{A-23}$$

Additionally,

$$\frac{dT_b}{dx} = \frac{dT_w}{dx} = \frac{\partial T}{\partial x} = \text{const.} \tag{A-24}$$

It can be shown that for fully developed heat transfer, $(T - T_w)$ is a function of r only. Thus, if one defines the dimensionless parameters

$$\theta = \frac{T - T_w}{\frac{q_w r_o}{k}} , \qquad \eta = \frac{r}{r_o} \tag{A-25}$$

and makes use of the relation u/U = 1, the energy equation (A-1) can be cast into the form

$$2\eta = \frac{d}{d\eta}\left(\frac{d\theta}{d\eta}\right). \tag{A-26}$$

The boundary conditions are

$$\eta = 1, \quad \theta = 0; \qquad \eta = 0, \quad \frac{d\theta}{d\eta} = 0. \tag{A-27}$$

The temperature solution is

$$\theta = \frac{T - T_w}{\frac{q_w r_o}{k}} = \frac{\eta^2}{2} - \frac{1}{2}. \tag{A-28}$$

From the definition of the bulk temperature, equation (A-11), one finds

$$\frac{T_b - T_w}{\frac{q_w r_o}{k}} = -\frac{1}{4}. \tag{A-29}$$

Thus, the Nusselt number based on tube radius is

$$Nu_{r_o} = \frac{hr_o}{k} = \frac{q_w}{T_w - T_b}\frac{r_o}{k} = 4. \tag{A-30}$$

Appendix B

TABLES OF NUMERICAL RESULTS

TABLE B-1

EIGENVALUES

(1) $L = 0.81$, $a\lambda_s/c_p = 0.1$

n	β_n				
1 - 5	2.640735	3.873923	6.055902	7.100608	9.474381
6 - 10	10.318789	12.860649	13.566449	16.185608	16.873978
11 - 15	19.442810	20.248398	22.654785	23.667191	25.843872
16 - 20	27.107925	29.022476	30.557770	32.197357	34.008987
21 - 25	35.373520	37.454529	38.556717	40.883850	41.756851
26 - 30	44.277435	44.993057	47.607910	48.292526	50.867401
31 - 35	51.662842	54.079361	55.080429	57.267578	58.521057
36 - 40	60.445190	61.971207	63.619141	65.422836	66.794418
41 - 45	68.868881	69.976761	72.298889	73.175919	75.693466
46 - 50	76.410919	79.025101	79.709030	82.285370	83.078415
51 - 55	85.497559	86.495605	88.685776	89.936142	91.863266

(2) $L = 0.81$, $a\lambda_s/c_p = 1.0$

n	β_n				
1 - 5	2.521017	4.049797	5.787669	7.412208	9.075620
6 - 10	10.746759	12.746759	14.068729	15.671379	17.383392
11 - 15	18.976547	20.692368	22.285980	23.996750	25.598999
16 - 20	27.297546	28.914688	30.595932	32.231949	33.893127
21 - 25	35.549622	37.190430	38.866455	40.488937	42.181412
26 - 30	43.789612	45.493668	47.093201	48.802750	50.400009
31 - 35	52.108459	53.710037	55.411118	57.022919	58.711197
36 - 40	60.337997	62.009567	63.654282	65.307220	66.970749
41 - 45	68.605179	70.286316	71.904510	73.600067	75.206009
46 - 50	76.911255	78.510300	80.219406	81.817657	83.524429
51 - 55	85.128098	86.826599	88.441177	90.126519	91.756226

TABLE B-1 (continued)

(3) L = 0.81, $a\lambda_s/c_p$ = 10.0

n	β_n				
1 - 5	2.424166	4.218837	5.567488	7.715923	8.738414
6 - 10	11.163369	11.934979	14.554319	15.181419	17.860519
11 - 15	18.509781	21.091599	21.911682	24.283737	25.351349
16 - 20	27.459198	28.806458	30.628082	32.266541	33.795685
21 - 25	35.725128	36.966446	39.174988	40.146759	42.602600
26 - 30	43.349670	45.980560	46.602386	49.278290	49.935287
31 - 35	52.506012	53.337997	55.697311	56.776718	58.872787
36 - 40	60.230545	62.041901	63.689392	65.209808	67.146881
41 - 45	68.380966	70.595627	71.561768	74.021988	74.765442
46 - 50	77.398361	78.019348	80.694595	81.353409	83.921509
51 - 55	84.756622	87.112549	88.195358	90.288010	91.649017

(4) L = 2.0, $a\lambda_s/c_p$ = 0.1

n	β_n				
1 - 5	-1.732839	3.646132	4.089878	6.085535	7.032230
6 - 10	8.353077	10.082729	10.654799	12.709249	13.383249
11 - 15	14.995629	16.434128	17.269645	19.293808	19.759949
16 - 20	21.638901	22.760178	23.906708	25.767075	26.244156
21 - 25	28.275085	29.084366	30.551025	32.132721	32.835037
26 - 30	34.886215	35.429855	37.196411	38.460068	39.465515
31 - 35	41.416595	41.854538	43.837509	44.779892	46.109078
36 - 40	47.818268	48.408830	50.462309	51.110504	52.753860
41 - 45	54.155029	55.026871	57.038345	57.490311	59.397751
46 - 50	60.475037	61.666016	63.491409	63.991501	66.032349
51 - 55	66.797897	68.311218	69.846207	70.591599	72.636292

TABLE B-1 (continued)

(5) $L = 2.0$, $a\lambda_s/c_p = 1.0$

n	β_n				
1 - 5	1.914936	3.285101	4.453687	5.909176	7.195330
6 - 10	8.442377	9.795597	10.984559	12.425679	13.603069
11 - 15	14.976179	16.282166	17.509048	18.935089	20.097610
16 - 20	21.509277	22.760788	24.040237	25.432205	26.601318
21 - 25	28.036270	29.238998	30.576080	31.917511	33.117111
26 - 30	34.552689	35.723389	37.112198	38.394836	39.643699
31 - 35	41.055908	42.218918	43.643707	44.869995	46.177979
36 - 40	47.545837	48.727890	50.165878	51.349426	52.714859
41 - 45	54.025528	55.249588	56.675308	57.838699	59.249268
46 - 50	60.500290	61.780899	63.170868	64.341217	65.776245
51 - 55	66.976608	68.317296	69.654327	70.857407	72.291611

(6) $L = 2.0$, $a\lambda_s/c_p = 10.0$

n	β_n				
1 - 5	2.239832	2.885285	4.832168	5.629963	7.177589
6 - 10	8.598171	9.503747	11.407169	12.026349	13.817869
11 - 15	14.942229	16.129669	17.877518	18.518051	20.435822
16 - 20	21.292038	22.761398	24.266541	25.090927	27.016006
21 - 25	27.674042	29.391998	30.619888	31.702911	33.520645
26 - 30	34.129608	36.013489	36.967468	38.329590	39.935776
31 - 35	40.674957	42.610168	43.333588	44.959732	46.299301
36 - 40	47.274429	49.149048	49.754929	51.585220	52.646881
41 - 45	53.896286	55.597595	56.266266	58.194199	59.002350
46 - 50	60.525528	61.976837	62.848511	64.761536	65.395797
51 - 55	67.153656	68.327896	69.463470	71.247940	71.869110

TABLE B-1 (continued)

(7) $L = 3.5$, $a\lambda_s/c_p = 0.1$

n	β_n				
1 - 5	1.318146	2.932016	3.824304	4.655346	6.273762
6 - 10	7.025402	8.001703	9.615089	10.204869	11.351259
11 - 15	12.946179	13.387099	14.702459	16.250092	16.593109
16 - 20	18.054382	19.500748	19.950769	21.406418	22.700058
21 - 25	23.159119	24.758011	25.874298	26.492722	28.108276
26 - 30	29.039169	29.836649	31.455856	32.201920	33.185226
31 - 35	34.797699	35.367935	36.536057	38.126099	38.546066
36 - 40	39.888000	41.421509	41.756577	43.240372	44.659088
41 - 45	45.024719	46.592606	47.849655	48.340088	49.944138
46 - 50	51.019699	51.676758	53.294067	54.182617	55.022141
51 - 55	56.640778	57.344849	58.371429	59.980499	60.512039

(8) $L = 3.5$, $a\lambda_s/c_p = 1.0$

n	β_n				
1 - 5	1.509260	2.815243	3.791039	4.828633	6.107348
6 - 10	7.068957	8.133332	9.399811	10.331989	11.439639
11 - 15	12.686279	13.594440	14.748099	15.964869	16.861328
16 - 20	18.057709	19.234970	20.135269	21.367187	22.497162
21 - 25	23.417221	24.675049	25.753052	26.707108	27.979782
26 - 30	29.005081	30.003967	31.279709	32.256119	33.306458
31 - 35	34.573288	35.509277	36.613022	37.859039	38.767288
36 - 40	39.922119	41.136032	42.032501	43.232239	44.404190
41 - 45	45.306229	46.541901	47.664429	48.588867	49.849579
46 - 50	50.918716	51.879761	53.153687	54.169678	55.177689
51 - 55	56.452576	57.420380	58.481110	59.744629	60.673889

TABLE B-1 (concluded)

(9) $L = 3.5$, $a\lambda_s/c_p = 10.0$

n	β_n				
1 - 5	1.869887	2.555204	3.757500	5.177959	5.774323
6 - 10	7.112193	8.434258	9.035312	10.456499	11.660569
11 - 15	12.326509	13.796149	14.868039	15.638579	17.130859
16 - 20	18.066589	18.963440	20.458298	21.264359	22.295792
21 - 25	23.774170	24.469070	25.632339	27.071869	27.689392
26 - 30	28.970978	30.344086	30.934052	32.310165	33.587738
31 - 35	34.207291	35.648560	36.807480	37.505569	38.984558
36 - 40	40.012222	40.821411	42.316071	43.210587	44.148148
41 - 45	45.639832	46.410202	47.481369	48.950699	49.618896
46 - 50	50.818192	52.241562	52.845566	54.156738	55.505630
51 - 55	56.098251	57.495529	58.741898	59.379128	60.833206

TABLE B-2

BULK TEMPERATURE AND MASS FRACTION DISTRIBUTIONS, $(T_b - T_f)/(T_o - T_f)$ and $(C_b - C_f)/(C_o - C_f)$

(1) $L = 0.81$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

$\frac{x/r_o}{Ur_o/D}$	$\frac{a\lambda_s}{c_p} = 0.1$	1.0	10.0
0	1.0	1.0	1.0
0.002	0.900796	0.906256	0.908763
0.004	0.862632	0.868321	0.873608
0.006	0.832949	0.939887	0.846229
0.008	0.808241	0.816107	0.823403
0.010	0.786680	0.795389	0.803464
0.012	0.767373	0.776834	0.785596
0.014	0.749795	0.759923	0.769310
0.016	0.733577	0.744317	0.754272
0.018	0.718472	0.729778	0.740255
0.020	0.704304	0.716132	0.727095
0.030	0.643595	0.657595	0.670579
0.040	0.594147	0.609824	0.624376
0.050	0.551930	0.568962	0.584787
0.060	0.514876	0.533031	0.549915
0.070	0.481756	0.500853	0.518633
0.080	0.451771	0.471666	0.490206
0.090	0.424367	0.444936	0.464126
0.100	0.399142	0.420279	0.440021
0.150	0.297159	0.319883	0.341243
0.200	0.223108	0.245827	0.267352
0.250	0.167952	0.189566	0.210245
0.300	0.126542	0.146373	0.165567
0.350	0.095372	0.113085	0.130452
0.400	0.071890	0.087390	0.102806
0.500	0.040855	0.052207	0.063863
0.600	0.023221	0.031195	0.039674
0.700	0.013199	0.018642	0.024648
0.800	0.007503	0.011140	0.015313
0.900	0.004265	0.006658	0.009513
1.000	0.002424	0.003979	0.005910

TABLE B-2 (continued)

(2) $L = 2.0$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

x/r_o / Ur_o/D	$\frac{a\lambda_s}{c_p} = 0.1$	1.0	10.0
0	1.0	1.0	1.0
0.002	0.897627	0.883707	0.865266
0.004	0.857509	0.838322	0.813007
0.006	0.826753	0.803611	0.773173
0.008	0.801137	0.774764	0.740169
0.010	0.778835	0.749699	0.711567
0.012	0.758885	0.727326	0.686098
0.014	0.740731	0.706992	0.663016
0.016	0.723990	0.688279	0.641818
0.018	0.708507	0.670890	0.622158
0.020	0.693791	0.654610	0.603802
0.030	0.631252	0.585268	0.526094
0.040	0.580416	0.529357	0.464107
0.050	0.537101	0.482109	0.412305
0.060	0.499156	0.441088	0.367856
0.070	0.465327	0.404851	0.329118
0.080	0.434770	0.372465	0.295000
0.090	0.406914	0.343281	0.264756
0.100	0.381344	0.316824	0.237832
0.150	0.278873	0.214944	0.140259
0.200	0.205707	0.147515	0.083377
0.250	0.152150	0.101756	0.049817
0.300	0.112635	0.070360	0.029874
0.350	0.083407	0.048707	0.017963
0.400	0.061770	0.033736	0.010822
0.500	0.033881	0.016196	0.003943
0.600	0.018584	0.007778	0.001441
0.700	0.010193	0.003735	0.000527
0.800	0.005591	0.001794	0.000193
0.900	0.003067	0.000862	0.000071
1.000	0.001682	0.000414	0.000026

TABLE B-2 (concluded)

(3) $L = 3.5$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

$\frac{x/r_o}{Ur_o/D}$	$\frac{a\lambda_s}{c_p} = 0.1$	1.0	10.0
0	1.0	1.0	1.0
0.002	0.896701	0.871954	0.831199
0.004	0.855325	0.820956	0.764941
0.006	0.824066	0.782668	0.715591
0.008	0.798072	0.750971	0.675029
0.010	0.775448	0.723495	0.640093
0.012	0.755220	0.699020	0.609180
0.014	0.736810	0.676826	0.581312
0.016	0.719840	0.656441	0.555866
0.018	0.704044	0.637538	0.532402
0.020	0.689240	0.619876	0.510614
0.030	0.625912	0.545124	0.419920
0.040	0.574554	0.485564	0.349946
0.050	0.530861	0.435942	0.293707
0.060	0.492670	0.393550	0.247653
0.070	0.458686	0.356759	0.209564
0.080	0.428061	0.324469	0.177877
0.090	0.400204	0.295885	0.151405
0.100	0.374683	0.270407	0.129212
0.150	0.272852	0.176128	0.060608
0.200	0.200521	0.116883	0.029851
0.250	0.147771	0.078122	0.015226
0.300	0.108988	0.052355	0.007950
0.350	0.080404	0.035122	0.004214
0.400	0.059321	0.023570	0.002254
0.500	0.032292	0.010619	0.000654
0.600	0.017579	0.004784	0.000192
0.700	0.009569	0.002156	0.000056
0.800	0.005209	0.000971	0.000017
0.900	0.002836	0.000438	0.000005
1.000	0.001544	0.000197	0.000001

TABLE B-3

WALL TEMPERATURE DISTRIBUTIONS $(T_w - T_f)/(T_o - T_f)$

(1) $L = 0.81$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

x/r_o / Ur_o/D	$\frac{a\lambda_s}{c_p} = 0.1$	1.0	10.0
0	------	------	------
0.002	-0.097634	-0.051342	-0.008950
0.004	-0.096410	-0.050762	-0.008856
0.006	-0.095474	-0.050306	-0.008780
0.008	-0.094678	-0.049913	-0.008715
0.010	-0.093969	-0.049560	-0.008656
0.012	-0.093318	-0.049234	-0.008602
0.014	-0.092711	-0.048930	-0.008551
0.016	-0.092138	-0.048642	-0.008503
0.018	-0.091592	-0.048367	-0.008457
0.020	-0.091068	-0.048103	-0.008413
0.030	-0.088683	-0.046897	-0.008210
0.040	-0.086543	-0.045812	-0.008028
0.050	-0.084544	-0.044798	-0.007857
0.060	-0.082634	-0.043828	-0.007694
0.070	-0.080780	-0.042884	-0.007535
0.080	-0.078958	-0.041957	-0.007378
0.090	-0.077149	-0.041035	-0.007222
0.100	-0.075338	-0.040112	-0.007066
0.150	-0.065905	-0.035291	-0.006250
0.200	-0.055764	-0.030089	-0.005366
0.250	-0.045689	-0.024897	-0.004480
0.300	-0.036521	-0.020144	-0.003664
0.350	-0.028687	-0.016048	-0.002956
0.400	-0.022260	-0.012655	-0.002364
0.500	-0.013102	-0.007729	-0.001490
0.600	-0.007581	-0.004663	-0.000931
0.700	-0.004349	-0.002798	-0.000579
0.800	-0.002483	-0.001676	-0.000360
0.900	-0.001415	-0.001002	-0.000224
1.0	-0.000806	-0.000599	-0.000139

TABLE B-3 (continued)

(2) $L = 2.0$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

$\frac{x/r_o}{Ur_o/D}$	$\frac{a\lambda_s}{c_p}$ = 0.1	1.0	10.0
0	-------	-------	-------
0.002	0.265813	0.166218	0.034983
0.004	0.262563	0.163850	0.034404
0.006	0.259907	0.161947	0.033944
0.008	0.257601	0.160304	0.033542
0.010	0.255491	0.158807	0.033180
0.012	0.253526	0.157417	0.032844
0.014	0.251670	0.156106	0.032528
0.016	0.249898	0.154857	0.032227
0.018	0.248194	0.153658	0.031940
0.020	0.246544	0.152499	0.031661
0.030	0.238854	0.147112	0.030373
0.040	0.231685	0.142116	0.029186
0.050	0.224722	0.137282	0.028043
0.060	0.217771	0.132474	0.026911
0.070	0.210702	0.127599	0.025769
0.080	0.203444	0.122609	0.024603
0.090	0.195977	0.117490	0.023412
0.100	0.188323	0.112260	0.022200
0.150	0.149299	0.085895	0.016182
0.200	0.113995	0.062644	0.011051
0.250	0.085489	0.044525	0.007243
0.300	0.063624	0.031242	0.004626
0.350	0.047209	0.021783	0.002906
0.400	0.034988	0.015141	0.001805
0.500	0.019198	0.007288	0.000682
0.600	0.010531	0.003502	0.000254
0.700	0.005776	0.001682	0.000094
0.800	0.003168	0.000808	0.000035
0.900	0.001738	0.000388	0.000013
1.000	0.000953	0.000186	0.000005

TABLE B-3 (concluded)

(3) $L = 3.5$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

$\frac{x/r_o}{Ur_o/D}$	$\frac{a\lambda_s}{c_p} = 0.1$	1.0	10.0
0	------	------	------
0.002	0.429162	0.292709	0.069975
0.004	0.423554	0.287902	0.068459
0.006	0.418973	0.284021	0.067244
0.008	0.414922	0.280610	0.066184
0.010	0.411202	0.277489	0.065218
0.012	0.407710	0.274569	0.064318
0.014	0.404385	0.271795	0.063466
0.016	0.401183	0.269132	0.062652
0.018	0.398078	0.266555	0.061867
0.020	0.395047	0.264045	0.061104
0.030	0.380518	0.252080	0.057502
0.040	0.366230	0.240411	0.054034
0.050	0.351598	0.228545	0.050547
0.060	0.336482	0.216372	0.047010
0.070	0.321005	0.204003	0.043458
0.080	0.305391	0.191627	0.039952
0.090	0.289864	0.179432	0.036549
0.100	0.274614	0.167574	0.033294
0.150	0.206117	0.116116	0.019955
0.200	0.152841	0.078840	0.011426
0.250	0.112937	0.053144	0.006389
0.300	0.083364	0.035728	0.003526
0.350	0.061515	0.023996	0.001932
0.400	0.045389	0.016111	0.001054
0.500	0.024709	0.007260	0.000312
0.600	0.013451	0.003271	0.000092
0.700	0.007322	0.001474	0.000027
0.800	0.003986	0.000664	0.000008
0.900	0.002170	0.000299	0.000002
1.000	0.001181	0.000135	0.000001

TABLE B-4.

WALL HEAT AND MASS FLUX DISTRIBUTIONS, $(q_w r_o)/[k(T_o - T_f)]$ and $\dot{m}/[\rho DL(C_f - C_o)]$

(1) $L = 0.81$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

$\frac{x/r_o}{Ur_o/D}$	$\frac{a\lambda_s}{c_p} = 0.1$	1.0	10.0
0	------	------	------
0.002	14.806780	14.197250	13.633500
0.004	10.288890	9.872953	9.487273
0.006	8.272547	7.953977	7.636926
0.008	7.087801	6.808466	6.550530
0.010	6.272922	6.028861	5.803305
0.012	5.669872	5.452227	5.250413
0.014	5.201179	5.003629	4.820708
0.016	4.822965	4.641870	4.474022
0.018	4.509387	4.342003	4.186671
0.020	4.243988	4.088140	3.943426
0.030	3.341618	3.225324	3.116857
0.040	2.801372	2.708995	2.622437
0.050	2.430912	2.355116	2.283752
0.060	2.155967	2.092643	2.032707
0.070	1.940977	1.887563	1.836699
0.080	1.766485	1.721268	1.677902
0.090	1.620812	1.582593	1.545630
0.100	1.496467	1.464385	1.433030
0.150	1.061122	1.052487	1.042506
0.200	0.784508	0.792295	0.797317
0.250	0.587615	0.606237	0.621333
0.300	0.441956	0.466635	0.487609
0.400	0.250820	0.278005	0.302207
0.500	0.142495	0.165974	0.187667
0.600	0.080979	0.099150	0.116578
0.700	0.046025	0.059244	0.072423
0.800	0.026261	0.035403	0.044993
0.900	0.014870	0.021157	0.027952
1.000	0.008453	0.012643	0.017366

TABLE B-4 (continued)

(2) $L = 2.0$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

$\frac{x/r_o}{Ur_o/D}$	$\frac{a\lambda_s}{c_p} = 0.1$	1.0	10.0
0	------	------	------
0.002	5.950845	6.740529	7.775357
0.004	4.297713	4.855691	5.580719
0.006	3.470428	3.912160	4.482295
0.008	2.968863	3.339863	3.815882
0.010	2.625671	2.948072	3.359521
0.012	2.372126	2.658486	3.022083
0.014	2.174940	2.433167	2.759419
0.016	2.015876	2.251330	2.547364
0.018	1.883997	2.210052	2.371401
0.020	1.772369	1.972776	2.222301
0.030	1.392701	1.537729	1.713719
0.040	1.165217	1.276206	1.406945
0.050	1.009059	1.095872	1.194392
0.060	0.893003	0.961027	1.034410
0.070	0.802096	0.854599	0.907129
0.080	0.728168	0.767342	0.801902
0.090	0.666332	0.693807	0.712580
0.100	0.613462	0.630567	0.635400
0.150	0.428091	0.409031	0.367024
0.200	0.311083	0.275308	0.215645
0.250	0.228980	0.188182	0.127780
0.300	0.169243	0.129547	0.076162
0.400	0.092747	0.061918	0.027377
0.500	0.050868	0.029703	0.009934
0.600	0.027901	0.014262	0.003623
0.700	0.015304	0.006849	0.001325
0.800	0.008394	0.003289	0.000485
0.900	0.004604	0.001580	0.000178
1.000	0.002526	0.000759	0.000065

TABLE B-4 (concluded)

(3) $L = 3.5$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

x/r_o / Ur_o/D	$\frac{a\lambda_s}{c_p} = 0.1$	1.0	10.0
0	------	------	------
0.002	3.596993	4.450274	5.807214
0.004	2.501697	3.071701	3.973777
0.006	2.013095	2.459828	3.158311
0.008	1.721243	2.094457	2.670962
0.010	1.521855	1.844681	2.337473
0.012	1.374545	1.659960	2.090565
0.014	1.259946	1.516108	1.898043
0.016	1.167484	1.399898	1.742303
0.018	1.090820	1.303417	1.612818
0.020	1.025900	1.221593	1.502817
0.030	0.804959	0.941709	1.124335
0.040	0.672332	0.771514	0.890689
0.050	0.581064	0.652477	0.724280
0.060	0.513112	0.562488	0.596702
0.070	0.459837	0.491251	0.495202
0.080	0.416538	0.433189	0.412945
0.090	0.380384	0.384906	0.345629
0.100	0.349552	0.344144	0.290220
0.150	0.242327	0.209588	0.126454
0.200	0.175376	0.135367	0.058724
0.250	0.128637	0.089536	0.028704
0.300	0.094740	0.059769	0.014564
0.400	0.051540	0.026853	0.004008
0.500	0.028054	0.012094	0.001151
0.600	0.015272	0.005449	0.000336
0.700	0.008313	0.002455	0.000098
0.800	0.004526	0.001106	0.000029
0.900	0.002464	0.000498	0.000009
1.000	0.001341	0.000225	0.000003

TABLE B-5

NUSSELT NUMBER DISTRIBUTIONS $(q_w r_o)/[k(T_b - T_w)]$

(1) $L = 0.81$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

$\frac{x/r_o}{Ur_o/D}$	$\frac{a\lambda_s}{c_p} = 0.1$	1.0	10.0
0	------	------	------
0.002	14.830059	14.825896	14.855953
0.004	10.728294	10.741194	10.750896
0.006	8.9103174	8.935115	8.931978
0.008	7.849864	7.861782	7.872115
0.010	7.123070	7.135179	7.145864
0.012	6.587579	6.600212	6.610964
0.014	6.173458	6.186081	6.197392
0.016	5.840958	5.853858	5.865458
0.018	5.566705	5.579942	5.591830
0.020	5.335854	5.349327	5.361506
0.030	4.563321	4.578229	4.591786
0.040	4.115488	4.131854	4.146774
0.050	3.819340	3.837192	3.853495
0.060	3.608250	3.627655	3.645399
0.070	3.450405	3.471457	3.490710
0.080	3.328415	3.351233	3.372094
0.090	3.231826	3.256559	3.279165
0.100	3.153910	3.180745	3.205255
0.150	2.922683	2.963302	3.000079
0.200	2.812149	2.871509	2.923595
0.300	2.710340	2.802332	2.881331
0.400	2.664028	2.778807	2.873510
0.500	2.640870	2.769190	2.871616
0.600	2.628977	2.765024	2.871036
0.700	2.622822	2.763194	2.870831
0.800	2.619617	2.762383	2.870760
0.900	2.617949	2.762023	2.870733
1.000	2.617080	2.761864	2.870723
∞	2.616134	2.761737	2.870716

TABLE B-5 (continued)

(2) $L = 2.0$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

$\frac{x/r_o}{Ur_o/D}$	$\frac{a\lambda_s}{c_p} =$ 0.1	1.0	10.0
0	------	------	------
0.002	9.418670	9.394611	9.364701
0.004	7.223694	7.199252	7.167735
0.006	6.122354	6.096904	6.063450
0.008	5.462125	5.435441	5.400139
0.010	5.017103	4.989189	4.952214
0.012	4.693942	4.664755	4.626197
0.014	4.447174	4.416821	4.376639
0.016	4.252075	4.220541	4.178812
0.018	4.093753	4.061083	4.017830
0.020	3.962842	2.928964	3.884184
0.030	3.549206	3.509551	3.457027
0.040	3.341386	3.295639	3.234942
0.050	3.230243	3.178036	3.109277
0.060	3.173599	3.114010	3.033949
0.070	3.150112	3.082387	2.990374
0.080	3.147802	3.071131	2.965649
0.090	3.158909	3.072777	2.952551
0.100	3.178211	3.082492	2.981588
0.150	3.303831	3.169587	2.958032
0.200	3.391969	3.243834	2.981538
0.300	3.453175	3.311716	3.016563
0.400	3.463105	3.329721	3.036305
0.500	3.464500	3.334140	3.046913
0.600	3.464685	3.335209	3.052505
0.700	3.464712	3.335461	3.055426
0.800	3.464718	3.335526	3.056932
0.900	3.464712	3.335541	3.057719
1.000	3.464713	3.335543	3.058124
∞	3.464717	3.335549	3.058558

TABLE B-5 (concluded)

(3) $L = 3.5$; $a\lambda_s/c_p = 0.1, 1.0, 10.0$

$\frac{x/r_o}{Ur_o/D}$	$\frac{a\lambda_s}{c_p} =$ 0.1	1.0	10.0
0	------	------	------
0.002	7.693464	7.682894	7.628783
0.004	5.794031	5.762456	5.705505
0.006	4.969458	4.933012	4.871335
0.008	4.492345	4.452868	4.386931
0.010	4.178092	4.136003	4.066050
0.012	3.955416	3.910843	3.836864
0.014	3.790165	3.743194	3.665264
0.016	3.663774	3.614423	3.532550
0.018	3.565168	3.513418	3.427624
0.020	3.487168	3.433070	3.343233
0.030	3.280279	3.213539	3.102320
0.040	3.227337	3.147063	3.009982
0.050	3.241405	3.146029	2.978611
0.060	3.285229	3.174707	2.973948
0.070	3.339891	3.215912	2.981236
0.080	3.395598	3.260916	2.993980
0.090	3.447386	3.305224	3.009249
0.100	3.493110	3.346620	3.025705
0.150	3.631150	3.492418	3.110610
0.200	3.678232	3.558249	3.187263
0.300	3.697334	3.594766	3.292069
0.400	3.699105	3.599992	3.340216
0.500	3.699259	3.600726	3.358827
0.600	3.699277	3.600818	3.365522
0.700	3.699277	3.600832	3.367873
0.800	3.699269	3.600832	3.368690
0.900	3.699270	3.600834	3.368975
1.000	3.699274	3.600830	3.369073
∞	3.699286	3.600840	3.369128

TABLE B-6

TEMPERATURE DISTRIBUTIONS $(T - T_f)/(T_o - T_f)$

(1) $L = 0.81$, $a\lambda_s/c_p = 0.1$

$\frac{r}{r_o}$ $\frac{x/r_o}{Ur_o/\alpha}$ =	0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.98604	0.83494	0.46316	0.23829	0.06025	0.00185
0.01	1.00000	0.98600	0.83484	0.46309	0.23826	0.06024	0.00185
0.02	1.00000	0.98590	0.83454	0.46286	0.23814	0.06021	0.00185
0.03	1.00000	0.98572	0.83404	0.46249	0.23794	0.06016	0.00185
0.04	1.00000	0.98548	0.83334	0.46197	0.23766	0.06008	0.00185
0.05	1.00000	0.98515	0.83244	0.46130	0.23729	0.05999	0.00184
0.10	1.00000	0.98241	0.82491	0.45571	0.23428	0.05921	0.00182
0.15	1.00000	0.97751	0.81231	0.44647	0.22931	0.05792	0.00178
0.20	1.00000	0.95997	0.79457	0.43367	0.22342	0.05614	0.00173
0.25	1.00000	0.95911	0.77161	0.41744	0.21372	0.05390	0.00166
0.30	1.00000	0.94408	0.74335	0.39797	0.20330	0.05121	0.00157
0.35	1.00000	0.92386	0.70977	0.37545	0.19130	0.04811	0.00148
0.40	1.00000	0.89732	0.67085	0.35014	0.17785	0.14466	0.00137
0.45	0.99999	0.86328	0.62669	0.32231	0.16312	0.04087	0.00125
0.50	0.99950	0.82060	0.57745	0.29227	0.14730	0.03682	0.00113
0.55	0.99795	0.76830	0.52342	0.26036	0.13057	0.03254	0.00100
0.60	0.99345	0.70569	0.46501	0.22695	0.11414	0.02810	0.00086
0.65	0.98190	0.63246	0.40276	0.19241	0.09521	0.02354	0.00072
0.70	0.95552	0.54885	0.33734	0.15715	0.07701	0.01892	0.00058
0.75	0.90220	0.45571	0.26955	0.12159	0.05876	0.01430	0.00044
0.80	0.80686	0.35451	0.20028	0.08613	0.04067	0.00974	0.00030
0.85	0.65616	0.24734	0.13053	0.05121	0.02295	0.00529	0.00016
0.90	0.44583	0.13680	0.06135	0.01723	0.00582	0.00100	0.00003
0.95	0.18698	0.02587	-0.00618	-0.01540	-0.01053	-0.00307	-0.00010
1.00	-0.09321	-0.08231	-0.07101	-0.04629	-0.02590	-0.00689	-0.00021

TABLE B-6 (continued)

(2) $L = 0.81$, $a\lambda_s/c_p = 1.0$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$	=0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.98649	0.84123	0.48109	0.25876	0.07306	0.00305
0.01	1.00000	0.98646	0.84114	0.48101	0.25872	0.07305	0.00305
0.02	1.00000	0.98636	0.84085	0.48080	0.25860	0.07302	0.00305
0.03	1.00000	0.98619	0.84036	0.48043	0.25840	0.07296	0.00304
0.04	1.00000	0.98595	0.83969	0.47992	0.25812	0.07288	0.00304
0.05	1.00000	0.98565	0.83882	0.47926	0.25775	0.07277	0.00304
0.10	1.00000	0.98301	0.83157	0.47381	0.25472	0.07191	0.00300
0.15	1.00000	0.97831	0.81943	0.46477	0.24971	0.07048	0.00294
0.20	1.00000	0.97108	0.80234	0.45226	0.24277	0.06850	0.00286
0.25	1.00000	0.96066	0.78021	0.43639	0.23399	0.06600	0.00275
0.30	1.00000	0.94623	0.75297	0.41732	0.22346	0.06300	0.00263
0.35	1.00000	0.92682	0.72058	0.39525	0.21131	0.05954	0.00248
0.40	1.00000	0.90133	0.68303	0.37043	0.19768	0.05566	0.00232
0.45	0.99987	0.86863	0.64040	0.34309	0.18272	0.05141	0.00214
0.50	0.99943	0.82762	0.59285	0.31355	0.16660	0.04683	0.00195
0.55	0.99795	0.77735	0.54063	0.28211	0.14951	0.04198	0.00175
0.60	0.99365	0.71713	0.48413	0.24912	0.13164	0.03692	0.00154
0.65	0.98259	0.64663	0.43386	0.21493	0.11320	0.03170	0.00132
0.70	0.95732	0.56619	0.36045	0.17994	0.09438	0.02638	0.00110
0.75	0.90622	0.47645	0.29465	0.14452	0.07541	0.02102	0.00088
0.80	0.81483	0.37887	0.22732	0.10906	0.05648	0.01569	0.00065
0.85	0.67035	0.27543	0.15940	0.07398	0.03783	0.01044	0.00043
0.90	0.46857	0.16861	0.09188	0.03966	0.01963	0.00532	0.00022
0.95	0.22007	0.06125	0.02577	0.00647	0.00210	0.00040	0.00002
1.00	-0.04918	-0.04366	-0.03790	-0.02521	-0.01458	-0.00427	-0.00018

TABLE B-6 (continued)

(3) $L = 0.81$, $a\lambda_s/c_p = 10.0$

$\frac{r}{r_o}$	$\frac{x/r_o}{Ur_o/\alpha}$ = 0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.98700	0.84710	0.19790	0.27827	0.08600	0.00456
0.01	1.00000	0.98698	0.84703	0.49784	0.27824	0.08599	0.00455
0.02	1.00000	0.98689	0.84675	0.49762	0.27812	0.08596	0.00455
0.03	1.00000	0.98672	0.84629	0.49727	0.27792	0.08589	0.00455
0.04	1.00000	0.98649	0.94564	0.49677	0.27763	0.08581	0.00454
0.05	1.00000.	0.98620	0.84480	0.49613	0.27727	0.08569	0.00454
0.10	1.00000	0.98367	0.83781	0.49079	0.27422	0.08475	0.00449
0.15	1.00000	0.97915	0.82610	0.48196	0.26919	0.08319	0.00441
0.20	1.00000	0.97219	0.80960	0.46972	0.26222	0.08103	0.00429
0.25	1.00000	0.96218	0.78824	0.45419	0.25338	0.07829	0.00415
0.30	1.00000	0.94831	0.76195	0.43552	0.24279	0.07501	0.00397
0.35	1.00000	0.92964	0.73067	0.41391	0.23054	0.07121	0.00377
0.40	0.99999	0.90512	0.69440	0.38955	0.21677	0.06695	0.00355
0.45	0.99988	0.87365	0.65320	0.36272	0.20163	0.06226	0.00330
0.50	0.99944	0.83417	0.60721	0.33367	0.18528	0.05721	0.00303
0.55	0.99802	0.78577	0.55668	0.30270	0.16791	0.05183	0.00275
0.60	0.99391	0.72776	0.50197	0.27015	0.14968	0.04620	0.00245
0.65	0.98329	0.65987	0.44355	0.23635	0.13081	0.04036	0.00214
0.70	0.95903	0.58226	0.38204	0.20166	0.11148	0.03439	0.00182
0.75	0.90997	0.49569	0.31813	0.16644	0.09190	0.02834	0.00150
0.80	0.82222	0.40148	0.25264	0.13107	0.07227	0.02228	0.00118
0.85	0.68342	0.30152	0.18646	0.09593	0.05280	0.01627	0.00086
0.90	0.48951	0.19818	0.12053	0.06127	0.03369	0.01037	0.00055
0.95	0.25056	0.09416	0.05583	0.02777	0.01513	0.00464	0.00025
1.00	-0.00859	-0.00767	-0.00669	-0.00453	-0.00270	-0.00086	-0.00005

TABLE B-6 (continued)

(4) $L = 2.0,\ a\lambda_s/c_p = 0.1$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$ =	0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.99033	0.88681	0.61838	0.43398	0.22904	0.05066
0.01	1.00000	0.99031	0.88675	0.61834	0.43395	0.22902	0.05066
0.02	1.00000	0.99024	0.88655	0.61817	0.43384	0.22897	0.05065
0.03	1.00000	0.99012	0.88620	0.61788	0.43366	0.22888	0.05063
0.04	1.00000	0.98996	0.88571	0.61748	0.43340	0.22876	0.05060
0.05	1.00000	0.98974	0.88509	0.61697	0.43307	0.22860	0.05057
0.10	1.00000	0.98789	0.87986	0.61271	0.43032	0.22729	0.05028
0.15	1.00000	0.98459	0.87110	0.60565	0.42576	0.22510	0.04981
0.20	1.00000	0.97950	0.85875	0.59584	0.41943	0.22206	0.04915
0.25	1.00000	0.97216	0.84274	0.58337	0.41137	0.21818	0.04831
0.30	1.00000	0.96198	0.82300	0.56833	0.40166	0.21349	0.04730
0.35	0.99990	0.94827	0.79948	0.55086	0.39036	0.20806	0.04611
0.40	0.99966	0.93024	0.77214	0.53108	0.37756	0.20179	0.04476
0.45	0.99961	0.90706	0.74100	0.50918	0.36335	0.19485	0.04325
0.50	0.99959	0.87793	0.70613	0.48532	0.34786	0.18724	0.04159
0.55	0.99875	0.84213	0.66770	0.45972	0.33118	0.17900	0.03979
0.60	0.99561	0.79914	0.62591	0.43257	0.31345	0.17018	0.03787
0.65	0.98759	0.74869	0.58109	0.40411	0.29479	0.16084	0.03582
0.70	0.96989	0.69083	0.53363	0.37457	0.27532	0.15102	0.03367
0.75	0.93449	0.62606	0.48402	0.34430	0.25520	0.14079	0.03142
0.80	0.87091	0.55526	0.43281	0.31324	0.23454	0.13021	0.02909
0.85	0.76970	0.47974	0.38061	0.28193	0.21349	0.11932	0.02669
0.90	0.62771	0.40116	0.32808	0.25052	0.19218	0.10820	0.02423
0.95	0.45226	0.32144	0.27590	0.21924	0.17074	0.09690	0.02173
1.00	0.26119	0.24260	0.22472	0.18832	0.14930	0.08549	0.01920

TABLE B-6 (continued)

(5) $L = 2.0,\ a\lambda_s/c_p = 1.0$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$ = 0.01		0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.98919	0.87229	0.57336	0.37434	0.17023	0.02671
0.01	1.00000	0.98917	0.87223	0.57332	0.37431	0.17021	0.02671
0.02	1.00000	0.98909	0.87200	0.57313	0.37420	0.17016	0.02670
0.03	1.00000	0.98896	0.87161	0.57281	0.37400	0.17008	0.02669
0.04	1.00000	0.98877	0.87106	0.57238	0.37373	0.16997	0.02667
0.05	1.00000	0.98852	0.87036	0.57181	0.37339	0.16983	0.02665
0.10	1.00000	0.98642	0.86447	0.56713	0.37049	0.16862	0.02647
0.15	1.00000	0.98267	0.85462	0.55932	0.36570	0.16661	0.02617
0.20	1.00000	0.97690	0.84074	0.54862	0.35905	0.16383	0.02574
0.25	1.00000	0.96858	0.82275	0.53495	0.35059	0.16028	0.02520
0.30	1.00000	0.95706	0.80058	0.51848	0.34042	0.15600	0.02455
0.35	1.00000	0.94153	0.77417	0.49936	0.32861	0.15102	0.12379
0.40	0.99973	0.92112	0.74351	0.47777	0.31527	0.14538	0.02293
0.45	0.99963	0.89490	0.80862	0.45389	0.30051	0.13911	0.02198
0.50	0.99957	0.86198	0.66960	0.42794	0.28446	0.13227	0.02093
0.55	0.99861	0.82155	0.62663	0.40015	0.26726	0.12490	0.01780
0.60	0.99508	0.77304	0.57998	0.37078	0.24905	0.11706	0.01859
0.65	0.98601	0.71616	0.53004	0.34009	0.22999	0.10881	0.01732
0.70	0.96591	0.65101	0.47725	0.30836	0.21022	0.10019	0.01599
0.75	0.92566	0.57817	0.42220	0.27588	0.18992	0.09129	0.01461
0.80	0.85338	0.49868	0.36551	0.24293	0.16924	0.08215	0.01319
0.85	0.73845	0.41405	0.30791	0.20982	0.14834	0.07284	0.01173
0.90	0.57744	0.32620	0.25016	0.17681	0.12738	0.06342	0.01026
0.95	0.37878	0.23731	0.19302	0.14420	0.10652	0.05396	0.00878
1.00	0.16286	0.14974	0.13728	0.11226	0.08589	0.04453	0.00729

TABLE B-6 (continued)

(6) $L = 2.0$, $a\lambda_s/c_p = 10.0$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$ = 0.01		0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.98752	0.85333	0.51627	0.30106	0.10442	0.00808
0.01	1.00000	0.98750	0.85326	0.51622	0.30103	0.10441	0.00807
0.02	1.00000	0.98741	0.85300	0.51601	0.30091	0.10437	0.00807
0.03	1.00000	0.98725	0.85255	0.51567	0.30071	0.10430	0.00807
0.04	1.00000	0.98703	0.85192	0.51518	0.30042	0.10420	0.00806
0.05	1.00000	0.98675	0.85112	0.51456	0.30006	0.10408	0.00805
0.10	1.00000	0.98433	0.84440	0.50938	0.29704	0.10306	0.00797
0.15	1.00000	0.98000	0.83315	0.50080	0.29204	0.10138	0.00785
0.20	1.00000	0.97335	0.81730	0.48890	0.28511	0.09904	0.00767
0.25	1.00000	0.96375	0.79677	0.47380	0.27633	0.09608	0.00745
0.30	1.00000	0.95046	0.77149	0.45564	0.26578	0.09252	0.00718
0.35	1.00000	0.93258	0.74140	0.43459	0.25357	0.08840	0.00687
0.40	0.99971	0.90908	0.70651	0.41087	0.23983	0.08375	0.00652
0.45	0.99960	0.87892	0.66685	0.38469	0.22469	0.07863	0.00614
0.50	0.99950	0.84107	0.62257	0.35634	0.20832	0.07328	0.00572
0.55	0.99838	0.79465	0.57388	0.32607	0.19088	0.06717	0.00527
0.60	0.99429	0.73900	0.52113	0.29421	0.17254	0.06094	0.00480
0.65	0.98380	0.67383	0.46477	0.26107	0.15348	0.05445	0.00431
0.70	0.96053	0.59930	0.40536	0.22698	0.13390	0.04778	0.00380
0.75	0.91390	0.51612	0.34357	0.19230	0.11399	0.04097	0.00328
0.80	0.83021	0.42552	0.28018	0.15738	0.09395	0.03410	0.00275
0.85	0.69730	0.32932	0.21602	0.12256	0.07396	0.02723	0.00223
0.90	0.51136	0.22975	0.15200	0.08820	0.05421	0.02043	0.00170
0.95	0.28239	0.12940	0.08904	0.05464	0.03489	0.01374	0.00119
1.00	0.03416	0.03100	0.02804	0.02220	0.01618	0.00724	0.00068

TABLE B-6 (continued)

(7) $L = 3.5$, $a\lambda_s/c_p = 0.1$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$ = 0.01		0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.99260	0.91174	0.69814	0.54600	0.36283	0.14905
0.01	1.00000	0.99259	0.91170	0.69810	0.54598	0.36281	0.14905
0.02	1.00000	0.99253	0.91154	0.69796	0.54589	0.36776	0.14903
0.03	1.00000	0.99244	0.91127	0.69773	0.54573	0.36268	0.14900
0.04	1.00000	0.99231	0.91089	0.69741	0.54551	0.36256	0.14895
0.05	1.00000	0.99214	0.91039	0.69700	0.54523	0.36240	0.14889
0.10	1.00000	0.99070	0.90629	0.69355	0.54291	0.36112	0.14841
0.15	1.00000	0.98813	0.89941	0.68783	0.53905	0.35900	0.14760
0.20	1.00000	0.98418	0.88971	0.67988	0.53369	0.35603	0.14647
0.25	1.00000	0.97847	0.87713	0.66976	0.52685	0.35225	0.14503
0.30	1.00000	0.97056	0.86160	0.65755	0.51860	0.34765	0.14327
0.35	1.00000	0.95989	0.84307	0.64333	0.50897	0.34227	0.14121
0.40	1.00000	0.94585	0.82152	0.62722	0.49805	0.33613	0.13885
0.45	0.99992	0.92780	0.79694	0.60933	0.48589	0.32925	0.13620
0.50	0.99951	0.90508	0.76938	0.58980	0.47258	0.32166	0.13327
0.55	0.99863	0.87714	0.73894	0.56879	0.45821	0.31340	0.13006
0.60	0.99641	0.84355	0.70578	0.54644	0.44286	0.30450	0.12659
0.65	0.99064	0.80407	0.67013	0.52293	0.42663	0.29499	0.12287
0.70	0.97719	0.75872	0.63228	0.49843	0.40961	0.28492	0.11892
0.75	0.94965	0.70786	0.59260	0.47312	0.39190	0.27433	0.11473
0.80	0.90016	0.65213	0.55151	0.44719	0.37361	0.26325	0.11034
0.85	0.82164	0.59253	0.50945	0.42082	0.35482	0.25174	0.10574
0.90	0.71165	0.53002	0.46693	0.39419	0.33565	0.23982	0.10097
0.95	0.57547	0.46691	0.42447	0.36747	0.31617	0.22755	0.09602
1.00	0.42657	0.40392	0.38256	0.34084	0.29649	0.21497	0.09092

TABLE B-6 (continued)

(8) $L = 3.5$, $a\lambda_s/c_p = 1.0$

$\frac{r}{r_o}$	$\frac{x/r_o}{Ur_o/\alpha}$ = 0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.99088	0.89134	0.63295	0.45573	0.26058	0.07935
0.01	1.00000	0.99086	0.89129	0.63291	0.45571	0.26057	0.07935
0.02	1.00000	0.99079	0.89109	0.63275	0.45560	0.26052	0.07934
0.03	1.00000	0.99068	0.80076	0.63247	0.45542	0.26043	0.07931
0.04	1.00000	0.99052	0.89029	0.63209	0.45518	0.26032	0.07928
0.05	1.00000	0.99031	0.88969	0.63159	0.45486	0.26017	0.07924
0.10	1.00000	0.98853	0.88466	0.62749	0.45223	0.25891	0.07890
0.15	1.00000	0.98535	0.87623	0.62069	0.44786	0.25683	0.07833
0.20	1.00000	0.98046	0.86435	0.61125	0.44180	0.25394	0.07754
0.25	1.00000	0.97341	0.84895	0.59924	0.43409	0.25024	0.07653
0.30	1.00000	0.96363	0.82996	0.58477	0.42479	0.24577	0.07531
0.35	1.00000	0.95045	0.80733	0.56795	0.41398	0.24055	0.07387
0.40	1.00000	0.93311	0.78102	0.54892	0.40174	0.23461	0.07223
0.45	0.99992	0.91083	0.75105	0.52784	0.38816	0.22799	0.07040
0.50	0.99957	0.88283	0.71750	0.50489	0.37336	0.22072	0.06838
0.55	0.99856	0.84842	0.68050	0.48025	0.35744	0.21284	0.06617
0.60	0.99572	0.80709	0.64027	0.45415	0.34052	0.20440	0.06380
0.65	0.98837	0.75857	0.59711	0.42678	0.32273	0.19545	0.06126
0.70	0.97147	0.70293	0.55141	0.39840	0.30419	0.18603	0.05858
0.75	0.93730	0.64062	0.50362	0.36923	0.28504	0.17619	0.05576
0.80	0.87598	0.57250	0.45429	0.33950	0.26540	0.16598	0.05281
0.85	0.77881	0.49982	0.40400	0.30947	0.24542	0.15547	0.04975
0.90	0.64267	0.42416	0.35338	0.27837	0.22521	0.14469	0.04659
0.95	0.47425	0.34737	0.30308	0.24943	0.20490	0.13371	0.04335
1.00	0.29047	0.27141	0.25375	0.21988	0.18462	0.12257	0.04003

TABLE B-6 (concluded)

(9) L = 3.5, $a\lambda_s/c_p$ = 10.0

$\frac{r}{r_o}$	$\frac{x/r_o}{Ur_o/\alpha}$ = 0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.98800	0.85852	0.53191	0.32154	0.12470	0.01599
0.01	1.00000	0.98798	0.85845	0.53186	0.32151	0.12469	0.01599
0.02	1.00000	0.98789	0.85820	0.53166	0.32139	0.12465	0.01598
0.03	1.00000	0.98774	0.85776	0.53132	0.32119	0.12458	0.01597
0.04	1.00000	0.98753	0.85716	0.53085	0.32091	0.12448	0.01596
0.05	1.00000	0.98726	0.85638	0.53024	0.32055	0.12435	0.01595
0.10	1.00000	0.98492	0.84989	0.52520	0.31758	0.12330	0.01584
0.15	1.00000	0.98075	0.83902	0.51686	0.31265	0.12155	0.01565
0.20	1.00000	0.97434	0.82370	0.50528	0.30583	0.11913	0.01539
0.25	1.00000	0.96509	0.80387	0.49058	0.29718	0.11606	0.01505
0.30	1.00000	0.95228	0.77943	0.47289	0.28678	0.11237	0.01465
0.35	1.00000	0.93504	0.75035	0.45239	0.27474	0.10808	0.01418
0.40	0.99999	0.91238	0.71661	0.42927	0.26118	0.10325	0.01365
0.45	0.99988	0.88329	0.67825	0.40376	0.24623	0.09791	0.01306
0.50	0.99946	0.84679	0.63540	0.37609	0.23004	0.09211	0.01242
0.55	0.99815	0.80200	0.58827	0.34655	0.21277	0.08590	0.01173
0.60	0.99437	0.74829	0.53719	0.31542	0.19459	0.07935	0.01099
0.65	0.98461	0.68538	0.48258	0.28301	0.17567	0.07250	0.01022
0.70	0.96225	0.61341	0.42498	0.24964	0.15619	0.06542	0.00941
0.75	0.91713	0.53303	0.36504	0.21565	0.13635	0.05817	0.00857
0.80	0.83630	0.44546	0.30349	0.18136	0.11632	0.05082	0.00771
0.85	0.70838	0.35240	0.24114	0.14713	0.09629	0.04342	0.00684
0.90	0.52952	0.25603	0.17886	0.11328	0.07644	0.03603	0.00595
0.95	0.30889	0.15880	0.11752	0.08014	0.05695	0.02872	0.00507
1.00	0.06927	0.06334	0.05800	0.04802	0.03799	0.02155	0.00418

TABLE B-7

MASS FRACTION DISTRIBUTIONS $(C - C_f)/(C_o - C_f)$

(1) $L = 0.81$, $a\lambda_s/c_p = 0.1$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$ = 0.01	0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.96730	0.76197	0.38822	0.19283	0.04768	0.00146
0.01	1.00000	0.96725	0.76188	0.38816	0.19280	0.04768	0.00146
0.02	1.00000	0.96709	0.76160	0.38800	0.19272	0.04766	0.00146
0.03	1.00000	0.96709	0.76113	0.38772	0.19259	0.04762	0.00146
0.04	1.00000	0.96645	0.76047	0.38724	0.19239	0.04758	0.00145
0.05	1.00000	0.96597	0.75962	0.38684	0.19215	0.04752	0.00145
0.10	1.00000	0.96188	0.75258	0.38275	0.19011	0.04701	0.00144
0.15	1.00000	0.95481	0.74087	0.37596	0.18673	0.04618	0.00141
0.20	1.00000	0.94438	0.72450	0.36656	0.18206	0.04503	0.00138
0.25	1.00000	0.93007	0.70353	0.35465	0.17613	0.04356	0.00133
0.30	1.00000	0.91126	0.67803	0.34034	0.16902	0.04181	0.00128
0.35	1.00000	0.88727	0.64811	0.32379	0.16079	0.03978	0.00122
0.40	0.99985	0.85740	0.61393	0.30516	0.15154	0.03750	0.00115
0.45	0.99936	0.82098	0.57568	0.28466	0.14136	0.03498	0.00107
0.50	0.99698	0.77743	0.53364	0.26248	0.13036	0.03227	0.00099
0.55	0.99443	0.72636	0.48813	0.23888	0.11865	0.02938	0.00090
0.60	0.98604	0.66760	0.43955	0.21408	0.10636	0.02634	0.00081
0.65	0.96808	0.60128	0.38835	0.18835	0.09360	0.02319	0.00071
0.70	0.93323	0.52787	0.33508	0.16195	0.08052	0.01996	0.00061
0.75	0.87201	0.44822	0.28031	0.13515	0.06725	0.01669	0.00051
0.80	0.77458	0.36353	0.22458	0.10822	0.05392	0.01339	0.00041
0.85	0.63427	0.27533	0.16885	0.08144	0.04065	0.01011	0.00031
0.90	0.45158	0.18539	0.11352	0.05508	0.02760	0.00688	0.00021
0.95	0.23681	0.09568	0.05938	0.02939	0.01487	0.00373	0.00011
1.00	0.00932	0.00823	0.00710	0.00463	0.00259	0.00069	0.00002

TABLE B-7 (continued)

(2) $L = 0.81,\ a\lambda_s/c_p = 1.0$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$ = 0.01		0.05	0.1	0.2	0.3	0.5	1.0
0	0.99999	0.96858	0.77103	0.40815	0.21348	0.05944	0.00247
0.01	0.99999	0.96852	0.77094	0.40810	0.21345	0.05943	0.00247
0.02	0.99999	0.96837	0.77066	0.40794	0.21337	0.05941	0.00247
0.03	0.99999	0.96812	0.77021	0.40767	0.21323	0.05937	0.00247
0.04	0.99999	0.96776	0.76958	0.40729	0.21303	0.05932	0.00247
0.05	0.99999	0.96729	0.76876	0.40681	0.21278	0.05925	0.00247
0.10	0.99999	0.96337	0.76197	0.40278	0.21070	0.05867	0.00244
0.15	0.99999	0.95659	0.75067	0.39612	0.20726	0.05773	0.00240
0.20	0.99999	0.94657	0.73488	0.38689	0.20249	0.05641	0.00235
0.25	0.99999	0.93283	0.71465	0.37518	0.19644	0.05474	0.00228
0.30	0.99998	0.91476	0.69004	0.36110	0.18916	0.05274	0.00220
0.35	0.99993	0.89172	0.66115	0.34480	0.18074	0.05042	0.00210
0.40	0.99979	0.86302	0.62813	0.32644	0.17125	0.04780	0.00199
0.45	0.99933	0.82802	0.59117	0.30620	0.16078	0.04491	0.00187
0.50	0.99802	0.78615	0.55052	0.28428	0.14945	0.04177	0.00174
0.55	0.99461	0.73703	0.50648	0.26091	0.13735	0.03843	0.00160
0.60	0.98657	0.68048	0.45943	0.23630	0.12460	0.03490	0.00145
0.65	0.96937	0.61664	0.40981	0.21070	0.11134	0.03122	0.00130
0.70	0.93598	0.54593	0.35811	0.18438	0.09767	0.02743	0.00114
0.75	0.87731	0.46916	0.30490	0.15757	0.08374	0.02356	0.00098
0.80	0.78389	0.38746	0.25076	0.13053	0.06967	0.01965	0.00082
0.85	0.64932	0.30228	0.19634	0.10353	0.05559	0.01573	0.00066
0.90	0.47403	0.21534	0.14229	0.07681	0.04164	0.01183	0.00049
0.95	0.26783	0.12849	0.08927	0.05062	0.02793	0.00800	0.00033
1.00	0.04918	0.04366	0.03790	0.02521	0.01458	0.00427	0.00018

TABLE B-7 (continued)

(3) $L = 0.81,\ a\lambda_s/c_p = 10.0$

$\frac{r}{r_o}$	$\frac{x/r_o}{Ur_o/\alpha}$ = 0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	0.99986	0.96967	0.77934	0.47682	0.23321	0.07142	0.00378
0.01	0.99986	0.96963	0.77927	0.42677	0.23319	0.07141	0.00378
0.02	0.99986	0.96948	0.77901	0.42662	0.23310	0.07139	0.00378
0.03	0.99986	0.96924	0.77857	0.42635	0.23296	0.07134	0.00377
0.04	0.99986	0.96889	0.77796	0.42598	0.23276	0.07128	0.00377
0.05	0.99986	0.96845	0.77717	0.42551	0.23251	0.07121	0.00377
0.10	0.99986	0.96468	0.77061	0.42156	0.23040	0.07057	0.00373
0.15	0.99986	0.95816	0.75970	0.41502	0.22691	0.06952	0.00368
0.20	0.99986	0.94852	0.74445	0.40595	0.22206	0.06806	0.00360
0.25	0.99986	0.93531	0.72491	0.39444	0.21591	0.06620	0.00350
0.30	0.99985	0.91793	0.70113	0.38060	0.20851	0.06396	0.00338
0.35	0.99982	0.89577	0.67321	0.36456	0.19993	0.06137	0.00325
0.40	0.99967	0.86815	0.64128	0.34647	0.19025	0.05844	0.00309
0.45	0.99923	0.83446	0.60553	0.32652	0.17955	0.05520	0.00292
0.50	0.99798	0.79415	0.56618	0.30488	0.16794	0.05167	0.00274
0.55	0.99472	0.74684	0.52353	0.28176	0.15552	0.04790	0.00254
0.60	0.98701	0.69236	0.47793	0.25738	0.14241	0.04392	0.00233
0.65	0.97050	0.63081	0.42979	0.23197	0.12871	0.03975	0.00210
0.70	0.93846	0.56262	0.37959	0.20576	0.11455	0.03543	0.00188
0.75	0.88213	0.48853	0.32785	0.17900	0.10006	0.03100	0.00164
0.80	0.79241	0.40962	0.27516	0.15193	0.08536	0.02651	0.00140
0.85	0.66313	0.32729	0.22209	0.12479	0.07058	0.02198	0.00116
0.90	0.49467	0.24315	0.16929	0.09782	0.05585	0.01746	0.00093
0.95	0.29638	0.15899	0.11736	0.07126	0.04128	0.01298	0.00069
1.00	0.08593	0.07666	0.06693	0.04533	0.02701	0.00858	0.00046

TABLE B-7 (continued)

(4) $L = 2.0$, $a\lambda_s/c_p = 0.1$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$ =	0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.99980	0.98665	0.84455	0.65407	0.36531	0.08157
0.01	1.00000	0.99980	0.98663	0.84448	0.65400	0.36527	0.08157
0.02	1.00000	0.99980	0.98653	0.84419	0.65373	0.36511	0.08153
0.03	1.00000	0.99979	0.98636	0.84372	0.65327	0.36484	0.08147
0.04	1.00000	0.99979	0.98613	0.84306	0.65264	0.36446	0.08138
0.05	1.00000	0.99978	0.98583	0.84221	0.65183	0.36397	0.08127
0.10	1.00000	0.99969	0.98326	0.83511	0.64505	0.35991	0.08036
0.15	1.00000	0.99949	0.97866	0.82323	0.63381	0.35318	0.07885
0.20	1.00000	0.99906	0.97160	0.80649	0.61818	0.34387	0.07675
0.25	1.00000	0.99821	0.96142	0.78481	0.59827	0.33208	0.07410
0.30	1.00000	0.99655	0.94732	0.75814	0.57424	0.31793	0.07092
0.35	1.00000	0.99345	0.92834	0.72641	0.54626	0.30157	0.06725
0.40	1.00000	0.98789	0.90343	0.68962	0.51458	0.28319	0.06313
0.45	1.00000	0.97836	0.87145	0.64784	0.47946	0.26297	0.05860
0.50	1.00000	0.96272	0.83135	0.60122	0.44122	0.24116	0.05372
0.55	0.99985	0.93822	0.78217	0.55002	0.40023	0.21797	0.04853
0.60	0.99964	0.90158	0.72326	0.49460	0.35688	0.19366	0.04310
0.65	0.99953	0.84931	0.65430	0.43546	0.31163	0.16850	0.03748
0.70	0.99688	0.77825	0.57552	0.37321	0.26496	0.14276	0.03174
0.75	0.98499	0.68620	0.48765	0.30858	0.21739	0.11670	0.02594
0.80	0.94745	0.57270	0.39206	0.24241	0.16945	0.09062	0.02013
0.85	0.85128	0.43953	0.29068	0.17560	0.12170	0.06479	0.01438
0.90	0.65649	0.29101	0.18593	0.10913	0.07468	0.03948	0.00875
0.95	0.34965	0.13370	0.08057	0.04399	0.02897	0.01495	0.00330
1.00	-0.02614	-0.02426	-0.02247	-0.01883	-0.01493	-0.00855	-0.00192

TABLE B-7 (continued)

(5) $L = 2.0,\ a\lambda_s/c_p = 1.0$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$ = 0.01		0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.99988	0.98499	0.82541	0.61475	0.30846	0.04992
0.01	1.00000	0.99988	0.98496	0.82533	0.61467	0.30842	0.04991
0.02	1.00000	0.99988	0.98485	0.82502	0.61438	0.30826	0.04988
0.03	1.00000	0.99987	0.98467	0.82409	0.61388	0.30798	0.04984
0.04	1.00000	0.99986	0.98440	0.82375	0.61319	0.30760	0.04978
0.05	1.00000	0.99985	0.98407	0.82281	0.61230	0.30710	0.04969
0.10	1.00000	0.99976	0.98116	0.81488	0.60489	0.30299	0.04901
0.15	1.00000	0.99953	0.97596	0.80162	0.59261	0.29619	0.04788
0.20	1.00000	0.99905	0.96798	0.78296	0.57554	0.28679	0.04633
0.25	1.00000	0.99808	0.95648	0.75882	0.55385	0.27492	0.04437
0.30	1.00000	0.99620	0.94057	0.72913	0.52769	0.26071	0.04202
0.35	1.00000	0.99268	0.91916	0.69387	0.49732	0.24436	0.03933
0.40	1.00000	0.98637	0.89108	0.65304	0.46301	0.22608	0.03632
0.45	1.00000	0.97556	0.85507	0.60676	0.42509	0.20609	0.03305
0.50	1.00000	0.95784	0.80995	0.55523	0.38396	0.18465	0.02954
0.55	0.99991	0.93010	0.75470	0.49877	0.34005	0.16204	0.02585
0.60	0.99963	0.88864	0.68859	0.43784	0.29387	0.13855	0.02284
0.65	0.99947	0.82955	0.61135	0.37304	0.24594	0.11447	0.01814
0.70	0.99647	0.74931	0.52325	0.30512	0.19687	0.09013	0.01420
0.75	0.98300	0.64550	0.42522	0.23494	0.14726	0.06583	0.01029
0.80	0.94034	0.51710	0.31887	0.16351	0.09776	0.04188	0.00645
0.85	0.83100	0.36805	0.20646	0.09191	0.04904	0.01859	0.00273
0.90	0.60981	0.20159	0.09080	0.02130	0.00176	-0.00376	-0.00083
0.95	0.26200	0.02589	-0.02490	-0.04715	-0.04343	-0.02488	-0.00418
1.00	-0.16286	-0.14974	-0.13730	-0.11226	-0.08589	-0.04452	-0.00729

TABLE B-7 (continued)

(6) $L = 2.0$, $a\lambda_s/c_p = 10.0$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$ =	0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.99989	0.98274	0.80083	0.56540	0.24165	0.02205
0.01	1.00000	0.99989	0.98271	0.80074	0.56532	0.24160	0.02204
0.02	1.00000	0.99989	0.98258	0.80039	0.56499	0.24144	0.02203
0.03	1.00000	0.99988	0.98236	0.79979	0.56445	0.24117	0.02200
0.04	1.00000	0.99987	0.98206	0.79895	0.56368	0.24079	0.02196
0.05	1.00000	0.99986	0.98167	0.79788	0.56270	0.24029	0.02191
0.10	1.00000	0.99975	0.97832	0.78892	0.55455	0.23622	0.02151
0.15	1.00000	0.99949	0.97236	0.77394	0.54104	0.22951	0.02084
0.20	1.00000	0.99893	0.96318	0.75287	0.52230	0.22025	0.01993
0.25	1.00000	0.99781	0.94998	0.72563	0.49852	0.20860	0.01878
0.30	1.00000	0.99564	0.93172	0.69219	0.46991	0.19473	0.01742
0.35	1.00000	0.99157	0.90718	0.65252	0.43679	0.17885	0.01587
0.40	1.00000	0.98429	0.87501	0.60670	0.39950	0.16122	0.01416
0.45	1.00000	0.97181	0.83382	0.55487	0.35846	0.14211	0.01233
0.50	1.00000	0.95138	0.78227	0.49733	0.31417	0.12183	0.01039
0.55	0.99991	0.91941	0.71923	0.43449	0.26717	0.10068	0.00840
0.60	0.99958	0.87168	0.64395	0.36693	0.21807	0.07901	0.00638
0.65	0.99936	0.80375	0.55617	0.29542	0.16756	0.05717	0.00437
0.70	0.99588	0.71161	0.45630	0.22086	0.11634	0.03549	0.00240
0.75	0.98032	0.59262	0.34552	0.14434	0.06518	0.01434	0.00050
0.80	0.93091	0.44643	0.22576	0.06708	0.01488	-0.00596	-0.00128
0.85	0.80427	0.27570	0.09975	-0.00960	-0.03376	-0.02508	-0.00293
0.90	0.54837	0.08643	-0.02918	-0.08430	-0.07995	-0.04771	-0.00442
0.95	0.14707	-0.11245	-0.15724	-0.25558	-0.12288	-0.05857	-0.00572
1.00	-0.34163	-0.30999	-0.28043	-0.22200	-0.16182	-0.07243	-0.00682

TABLE B-7 (continued)

(7) $L = 3.5$, $a\lambda_s/c_p = 0.1$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$ = 0.01		0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.99980	0.99953	0.97492	0.89493	0.67619	0.28984
0.01	1.00000	0.99980	0.99953	0.97488	0.89487	0.67612	0.28981
0.02	1.00000	0.99980	0.99952	0.97474	0.89461	0.67584	0.28967
0.03	1.00000	0.99980	0.99951	0.97451	0.89417	0.67537	0.28946
0.04	1.00000	0.99980	0.99949	0.97417	0.89356	0.67472	0.28915
0.05	1.00000	0.99980	0.99947	0.97374	0.89277	0.67388	0.28875
0.10	1.00000	0.99980	0.99925	0.97009	0.88617	0.66687	0.28547
0.15	1.00000	0.99980	0.99879	0.96370	0.87505	0.65524	0.28005
0.20	1.00000	0.99980	0.99789	0.95415	0.85924	0.63905	0.27254
0.25	1.00000	0.99980	0.99622	0.94083	0.83852	0.61842	0.26302
0.30	1.00000	0.99980	0.99325	0.92304	0.81265	0.59350	0.25162
0.35	1.00000	0.99969	0.98814	0.89995	0.78139	0.56444	0.23845
0.40	1.00000	0.99927	0.97969	0.87069	0.74458	0.53149	0.22365
0.45	1.00000	0.99815	0.96620	0.83441	0.70208	0.49492	0.20741
0.50	1.00000	0.99535	0.94553	0.79034	0.65389	0.45506	0.18989
0.55	1.00000	0.98897	0.91508	0.73790	0.60013	0.41227	0.17129
0.60	1.00000	0.97566	0.87201	0.67676	0.54110	0.36700	0.15182
0.65	1.00000	0.95010	0.81356	0.60694	0.47727	0.31969	0.13169
0.70	0.99995	0.90500	0.73749	0.52889	0.40928	0.27087	0.11112
0.75	0.99871	0.83197	0.64260	0.44347	0.33797	0.22110	0.09034
0.80	0.99045	0.72343	0.52922	0.35205	0.26433	0.17095	0.06957
0.85	0.94668	0.57549	0.39955	0.25637	0.18950	0.12101	0.04902
0.90	0.79549	0.39078	0.25771	0.15855	0.11470	0.07190	0.02893
0.95	0.45565	0.17975	0.10949	0.06092	0.04122	-0.02420	-0.00949
1.00	-0.04266	-0.04039	-0.03826	-0.02308	-0.02965	-0.02150	-0.00909

TABLE B-7 (continued)

(8) $L = 3.5,\ a\lambda_s/c_p = 1.0$

$\frac{r}{r_o}$ \ $\frac{x/r_o}{Ur_o/\alpha}$ =	0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	0.99998	0.99959	0.96929	0.87206	0.61369	0.20536
0.01	1.00000	0.99998	0.99958	0.96925	0.87198	0.61369	0.20536
0.02	1.00000	0.99998	0.99957	0.96908	0.87167	0.61328	0.20521
0.03	1.00000	0.99998	0.99956	0 96879	0.87114	0.61274	0.20501
0.04	1.00000	0.99998	0.99954	0.96838	0.87040	0.61199	0.20472
0.05	1.00000	0.99998	0.99951	0.96785	0.86945	0.61102	0.20436
0.10	1.00000	0.99998	0.99924	0.96338	0.86149	0.60296	0.20133
0.15	1.00000	0.99998	0.99866	0.95556	0.84809	0.58961	0.19632
0.20	1.00000	0.99998	0.99753	0.94387	0.82905	0.57107	0.18942
0.25	1.00000	0.99996	0.99546	0.92761	0.80413	0.54750	0.18073
0.30	1.00000	0.99991	0.99178	0.90590	0.77308	0.51912	0.17036
0.35	1.00000	0.99972	0.98547	0.87777	0.73567	0.48619	0.15846
0.40	1.00000	0.99920	0.97502	0.84219	0.69172	0.44905	0.14522
0.45	1.00000	0.99778	0.95840	0.79816	0.64116	0.40810	0.13083
0.50	1.00000	0.99430	0.93296	0.74481	0.58405	0.36380	0.11548
0.55	1.00000	0.98640	0.89553	0.68156	0.52065	0.31667	0.09940
0.60	1.00000	0.96991	0.84268	0.60792	0.45141	0.26730	0.08283
0.65	1.00000	0.93830	0.77109	0.52424	0.37704	0.21635	0.06599
0.70	0.99987	0.88261	0.67813	0.43112	0.29844	0.16452	0.04913
0.75	0.99856	0.79255	0.56249	0.32980	0.21679	0.11254	0.03247
0.80	0.98816	0.65898	0.42481	0.22209	0.13345	0.06119	0.01625
0.85	0.93352	0.47740	0.26803	0.11036	0.04995	0.01125	0.00070
0.90	0.74596	0.25147	0.09752	-0.00261	-0.03204	-0.03652	-0.01400
0.95	0.32500	-0.00538	-0.07931	-0.11376	-0.11080	-0.08136	-0.02762
1.00	-0.29047	-0.27140	-0.25375	-0.21988	-0.18462	-0.12257	-0.04003

TABLE B-7 (concluded)

(9) L = 3.5, $a\lambda_s/c_p$ = 10.0

$\frac{x/r_o}{Ur_o/\alpha}$ =	0.01	0.05	0.1	0.2	0.3	0.5	1.0
0	1.00000	1.00000	0.99948	0.96018	0.83634	0.52196	0.10836
01	1.00000	1.00000	0.99948	0.96013	0.83624	0.52187	0.10834
02	1.00000	1.00000	0.99947	0.95990	0.83585	0.52149	0.10824
03	1.00000	1.00000	0.99945	0.95953	0.83518	0.52087	0.10808
04	1.00000	1.00000	0.99942	0.95900	0.83425	0.51999	0.10786
05	1.00000	1.00000	0.99938	0.95833	0.83306	0.51886	0.10758
10	1.00000	1.00000	0.99902	0.95256	0.82304	0.50948	0.10521
15	1.00000	1.00000	0.99826	0 94249	0.80619	0.49398	0.10132
.20	1.00000	1.00000	0.99679	0.92747	0.78230	0.47253	0.09599
.25	1.00000	0.99997	0.99407	0.90659	0.75111	0.44539	0.08933
.30	1.00000	0.99989	0.98926	0.87878	0.71237	0.41291	0.08148
.35	1.00000	0.99965	0.98101	0.84283	0.66587	0.37551	0.07261
.40	1.00000	0.99895	0.96740	0.79748	0.61149	0.33372	0.06291
.45	1.00000	0.99708	0.94576	0.74155	0.54926	0.28814	0.05259
.50	1.00000	0.99250	0.91269	0.67403	0.47945	0.23948	0.04183
.55	1.00000	0.98212	0.86417	0.59427	0.40253	0.18853	0.03089
.60	1.00000	0.96050	0.79582	0.50207	0.31932	0.13615	0.01999
.65	0.99996	0.91910	0.70351	0.39787	0.23090	0.08328	0.00935
.70	0.99983	0.84632	0.58409	0.28280	0.13873	0.03093	-0.00082
.75	0.99814	0.72894	0.43621	0.15877	0.04454	-0.01988	-0.01030
.80	0.98435	0.55537	0.26113	0.02846	-0.04965	-0.06811	-0.01891
.85	0.91207	0.33042	0.06319	-0.10473	-0.14160	-0.11272	-0.02650
.90	0.66475	0.02975	-0.15006	-0.23684	-0.22891	-0.15274	-0.03291
.95	0.11145	-0.29803	-0.36842	-0.36351	-0.30915	-0.18725	-0.03805
1.00	-0.69268	-0.63347	-0.58002	-0.48023	-0.37991	-0.21545	-0.04183

Appendix C

COMPUTER PROGRAM

The computer program used in obtaining the numerical results is given in this Appendix. The program was written in Fortran IV language and an IBM 360 Model 50 Computer was used to obtain the results.

```
      COMBINED HEAT AND MASS TRANSFER IN A CIRCULAR TUBE
      SLUG FLOW PROFILE
      DIMENSION  BETA(100),AN(100),BN(100),TB(100),TW(100),
     1YN(50),XN(50),BES1(100),BES0(100),BE0C(100),BE1C(100),
     2QW(100)
      READ (1,201) NX,NY
  201 FORMAT(2I10)
      READ (1,200) (XN(I),I=1,NX)
      READ (1,200) (YN(I),I=1,NY)
  200 FORMAT(10F8.0)
      DO 49 J=1,9
      READ (1,100) A,B,N
  100 FORMAT(2F10.2,I10)
      WRITE(3,101) A,B,N
  101 FORMAT('1',6X,'A =',F10.2,6X,'B =',F10.2,6X,'N =',I3/)
      READ (1,102)(BETA(I),I=1,N)
  102 FORMAT(7F10.6)
      WRITE (3,103)
  103 FORMAT(/6X,'EIGENVALUES'//)
      WRITE (3,121) (BETA(I),I=1,N)
  121 FORMAT (6X,8F12.6)
      WRITE (3,104)
  104 FORMAT ('1',10X,'AN',18X,'AN',18X,'AN',18X,'AN',18X,
     1'AN'////)
C     TO CALCULATE SERIES COEFFICIENT AN AND BN
      EPS=1.0E-06
      DO 1 I=1,N
      X=BETA(I)
      XI=X*SQRT(B)
      CALL BESJ (X,0,B0,EPS,IER0)
      CALL BESJ (X,1,B1,EPS,IER1)
      CALL BESJ (XI,0,B0C,EPS,IER0C)
      CALL BESJ (XI,1,B1C,EPS,IER1C)
      BES0(I)=B0
      BES1(I)=B1
      BE0C(I)=B0C
      BE1C(I)=B1C
      AN1=B1*(1.0+A)/(X*A*A)
      AN2=(B0*B0+B1*B1)*0.5/A
      AN3=(B0C*B0C+B1C*B1C )*0.5*B0*B0/(B0C*B0C)
      AN(I)=AN1/(AN2+AN3)
      BN(I)=-AN(I)*A*B0/B0C
    1 CONTINUE
      WRITE (3,105) (AN(I),I=1,N)
  105 FORMAT(5E20.7)
      WRITE (3,106)
  106 FORMAT(/10X,'BN',18X,'BN',18X,'BN',18X,'BN',18X,'BN'/)
      WRITE (3,105) (BN(I),I=1,N)
C     BULK TEMPERATURE AND BULK MASS FRACTION CALCULATION
      WRITE (3,301) A,B
  301 FORMAT('1',6X,'A =',F8.4,5X,'B =',F8.4///)
      WRITE (3,107)
  107 FORMAT(7X,'I',10X,'K',9X,'DISTANCE X',9X,'DISTANCE KAI'
     1,15X,'TBULK'//)
      XX=1.3
```

```
      DO 2 I=1,43
      XL=XX*B
      SUM=0.0
      K=1
   60 BB=BETA(K)
      A1=AN(K)
      B2=BB*BB
      BES=BES1(K)
      IF(I.GE. 43) GO TO 62
   61 PSB=A1*BES/(BB*EXP(B2*XL))
      GO TO 63
   62 PSB=A1*BES/BB
      XX=0.0
      XL=0.0
   63 PSUM=ABS(SUM)
      SUM=SUM+PSB
      QSUM=ABS(SUM)
      CNVG=ABS(QSUM-PSUM)/QSUM
      IF(CNVG .LE. 1.0E-06) GO TO 64
      K=K+1
      IF (K .LE. N)  GO TO 60
   64 TBULK=SUM*2.
      TB(I)=TBULK
      WRITE(3,108)I,K,XX,XL,TBULK
  108 FORMAT(2I10,2E20.4,E20.7)
      IF (I .GE. 25) GO TO 152
  151 XX=XX-0.05
      GO TO 2
  152 IF (I .GE. 33) GO TO 154
  153 XX=XX-0.01
      GO TO 2
  154 XX=XX-0.002
    2 CONTINUE
C     WALL TEMPERATURE CALCULATION
      WRITE (3,301) A,B
      WRITE (3,110)
  110 FORMAT(7X,'I',10X,'K',9X,'DISTANCE X',8X,'DISTANCE KAI'
     1,10X,'TWALL'//)
      XX=1.3
      DO 4 I=1,43
      XL=XX*B
      SUM=0.0
      K=1
   76 BB=BETA(K)
      A1=AN(K)
      B2=BB*BB
      BES=BES0(K)
      IF(I-43)77,78,78
   77 PS=A1*BES/EXP(B2*XL)
      GO TO 79
   78 PS=A1*BES
      XX=0.0
      XL=0.0
   79 PSUM=ABS(SUM)
      SUM=SUM+PS
```

```
      QSUM=ABS(SUM)
      CNVG=ABS(QSUM-PSUM)/QSUM
      IF (CNVG .LE. 1.0E-06) GO TO 80
      K=K+1
      IF(K-N) 76,76,80
   80 TWALL=SUM
      TW(I)=TWALL
      WRITE(3,108) I,K,XX,XL,TWALL
      IF (I .GE. 25) GO TO 161
  160 XX=XX-0.05
      GO TO 4
  161 IF(I .GE. 33) GO TO 163
  162 XX=XX-0.01
      GO TO 4
  163 XX=XX-0.002
    4 CONTINUE
C     WALL TEMPERATURES VERY NEAR ENTRANCE OF THE TUBE
      WRITE(3,122)A,B
  122 FORMAT ('1',6X,'WALL TEMPERATURES AT SMALL VALUES OF
     1KAI(XL)',5X,'A =',F8.4,5X,'B =',F8.4,///)
      XL=0.02
      DO 11 I=1,20
      XX=XL/B
      SUM=0.0
      K=1
  180 BB=BETA(K)
      A1=AN(K)
      B2=BB*BB
      BES=BESO(K)
      IF(I .GE. 20) GO TO 181
      PS=A1*BES/EXP(B2*XL)
      GO TO 182
  181 PS=A1*BES
      XX=0.0
      XL=0.0
  182 PSUM=ABS(SUM)
      SUM=SUM+PS
      QSUM=ABS(SUM)
      CNVG=ABS(QSUM-PSUM)/QSUM
      IF (CNVG .LE. 1.0E-06) GO TO 183
      K=K+1
      IF (K .LE. N) GO TO 180
  183 TWALL=SUM
      WRITE (3,123) I,K,XX,XL,TWALL
  123 FORMAT (2I10,2E20.4,E20.6)
      XL=XL-0.001
   11 CONTINUE
C     WALL HEAT FLUX AND MASS FRACTION CALCULATION
      WRITE (3,111) A,B
  111 FORMAT('1','WALL HEAT FLUX VS. AXIAL DISTANCE',5X,
     1'A =',F8.4,5X,'B =',F8.4///)
      WRITE (3,112)
  112 FORMAT(7X,'I',10X,'K',9X,'DISTANCE X',9X,'DISTANCE KAI'
     1,10X,'QWALL'//)
      XX=1.3
```

```
      DO 5 I=1,43
      XL=XX*B
      SUM=0.0
      K=1
   81 BB=BETA(K)
      A1=AN(K)
      B2=BB*BB
      IF(I .GE. 43) GO TO 83
   82 PS=A1*BB*BES1(K)/EXP(B2*XL)
      GO TO 84
   83 PS=A1*BB*BES1(K)
      XX=0.0
      XL=0.0
   84 PSUM=ABS(SUM)
      SUM=SUM+PS
      QSUM=ABS(SUM)
      CNVG=ABS(QSUM-PSUM)/QSUM
      IF(CNVG .LE.1.0E-06) GO TO 85
      K=K+1
      IF(K .LE. N) GO TO 81
   85 QWALL=SUM
      QW(I)=QWALL
      WRITE (3,108) I,K,XX,XL,QWALL
      IF (I .GE. 25) GO TO 166
  165 XX=XX-0.05
      GO TO 5
  166 IF (I .GE. 33) GO TO 168
  167 XX=XX-0.01
      GO TO 5
  168 XX=XX-0.002
    5 CONTINUE
C     NUSSELT NUMBER (BASED ON TUBE RADIUS)
      WRITE (3,301) A,B
      WRITE (3,113)
  113 FORMAT(6X,'NUSSELT NUMBER VS.AXIAL DISTANCE'///)
      BB=BETA(1)
      XNFD=(BB*BES1(1))/(2*BES1(1)/BB-BES0(1))
      WRITE (3,114) XNFD
  114 FORMAT(6X,'FULLY-DEVELOPED NUSSELT NUMBER =',F10.6///)
      WRITE (3,115)
  115 FORMAT(7X,'I',9X,'DISTANCE X',9X,'DISTANCE KAI',10X,
     1'NUSSELT NO.'//)
      XX=1.3
      DO 6 I=1,43
      XL=XX*B
      XNU=QW(I)/(TB(I)-TW(I))
      WRITE (3,116) I,XX,XL,XNU
  116 FORMAT (I10,2E20.4,F20.8)
      IF(I .GE. 25) GO TO 171
  170 XX=XX-0.05
      GO TO 6
  171 IF (I .GE. 33) GO TO 173
  172 XX=XX-0.01
      GO TO 6
  173 XX=XX-0.002
```

```
    6 CONTINUE
C     TEMPERATURE DISTRIBUTION CALCULATION
      DO 7 I=1,NX
      XL=XN(I)
      WRITE (3,117)A,B,XL
  117 FORMAT ('1',6X,'TEMPERATURE DISTRIBUTION',5X,'A =',
     1F8.4,5X,'B =',F8.4///6X,'XL =',F8.4///)
      DO 8 L=1,NY
      SUM=0.0
      K=1
  175 BB=BETA(K)
      A1=AN(K)
      B2=BB*BB
      Y=BB*YN(L)
      IF (L .EQ. 1) GO TO 179
      CALL BESJ(Y,0,B0,EPS,IERO)
      PS=A1*B0/EXP(B2*XL)
      GO TO 180
  179 PS=A1/EXP(B2*XL)
  180 PSUM=ABS(SUM)
      SUM=SUM+PS
      QSUM=ABS(SUM)
      CNVG=ABS(QSUM-PSUM)/QSUM
      IF(CNVG .LE.1.0E-06) GO TO 176
      K=K+1
      IF (K .LE. N) GO TO 175
  176 TEMP=SUM
      WRITE (3,118) L, K, YN(L),TEMP
  118 FORMAT (2I10,F20.4,F20.6)
    8 CONTINUE
    7 CONTINUE
C     CALCULATION OF CONCENTRATION DISTRIBUTION
      DO 9 I=1,NX
      XL=XN(I)
      WRITE (3,119) A,B,XL
  119 FORMAT('1',6X,'CONCENTRATION DISTRIBUTION',5X,'A =',
     1F8.4,5X,'B =',F8.4///6X,'XL =',F8.4///)
      DO 10 L=1,NY
      SUM=0.0
      K=1
  177 BB=BETA(K)
      B3=BB*SQRT(B)
      B2=BB*BB
      B1=BN(K)
      Y=B3*YN(L)
      IF (L .EQ. 1) GO TO 181
      CALL BESJ (Y,0,B0,EPS,IERO)
      PS=B1*B0/EXP(B2*XL)
      GO TO 182
  181 PS=B1/EXP(B2*XL)
  182 PSUM=ABS(SUM)
      SUM=SUM+PS
      QSUM=ABS(SUM)
      CNVG=ABS(QSUM-PSUM)/QSUM
      IF( CNVG .LE. 1.0E-06) GO TO 178
```

```
      K=K+1
      IF(K .LE. N) GO TO 177
  178 CONC=SUM
      WRITE(3,120) L,K,YN(L),CONC
  120 FORMAT (2I10,E20.4,E20.6)
   10 CONTINUE
    9 CONTINUE
   49 CONTINUE
      STOP
      END
```